Projection Operators in Computable Analysis

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Projecting an arbitrary point x over a non-empty given subset of the Euclidean space is an example of a mathematical problem deeply grounded in our geometrical intuition of the spatial continuum that has important applications in higher mathematics. The question that I address in my talk is then the following: does the intuitive, even empirical, naturalness of this problem correspond to an algorithmic simplicity of solution? The problem of finding points in a closed set A of minimal distance with respect to a given point x has been investigated already by other authors and proved to be computable for some types of metric spaces (and closed sets representations) in presence of some optimal conditions. But what does it happen when such optimal conditions may fail to hold? It is not very surprising that this problem is then no longer computably solvable, but I will classify the degrees of incomputability in the Weihrauch lattice of the projection operators defined by the different usual types of information for closed sets. Beside the projection operators of exact precision, I will also consider corresponding approximated versions, to determine whether they are really computably simpler, as the intuition suggests. It turns out that such operators are useful to characterize some Weihrauch degrees of fundamental importance. The approximated projection operators with total information for closed sets are even computable. Still, they might be seen of no practical importance, unless concrete examples of applications are shown. I will then analyze the classical Whitney Extension Theorem as a case study. This is a joint work with Alberto Marcone improving some preliminary results presented at CCA in Faro.