Relativizations of the P =? DNP Question for the BSS Model

Christine Gaßner
Greifswald

The machines

Computation instructions:

$$l: \ Z_i := Z_j \circ Z_k, \qquad \circ \in \{+, -, \cdot\},$$

$$l: \ Z_j := c,$$

Branching instructions:

$$l: \ \text{if } Z_j = 0 \text{ then goto } l_1 \text{ else goto } l_2,$$

$$l: \ \text{if } Z_j \geq 0 \text{ then goto } l_1 \text{ else goto } l_2,$$

Copy instructions:

$$l: \ Z_{I_j} := Z_{I_k},$$

Index instructions:

$$l: \ I_j := 1,$$

$$l: \ I_j := I_j + 1,$$

$$l: \ \text{if } I_j = I_k \text{ then goto } l_1 \text{ else goto } l_2.$$

The complexity classes

The input: $\mathbf{x} = (x_1, \ldots, x_n) \in \bigcup_{i \geq 1} \mathbb{R}^i$.

The guesses: $\mathbf{y} = (y_1, \ldots, y_m) \in \bigcup_{i \geq 1} \mathbb{R}^i$.

Assignment: $\mathbf{x} \mapsto Z_1, \ldots, Z_n \quad \mathbf{y} \mapsto Z_{n+1}, \ldots, Z_{n+m} \quad n \mapsto I_1.$

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Polynomial time: $\text{cost}(\mathbf{x}) \leq kn^c$.

size($\mathbf{x}$): n.
cost($\mathbf{x}$): Number of executed instructions on $\mathbf{x}$.

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$$\underbrace{y_1, \ldots, y_m = 0}_{\Downarrow \atop \mathrm{P}_{\mathbb{R}}} \qquad \underbrace{y_1, \ldots, y_m \in \{0, 1\}}_{\Downarrow \atop \mathrm{DNP}_{\mathbb{R}}} \qquad \underbrace{y_1, \ldots, y_m \in \mathbb{R}}_{\Downarrow \atop \mathrm{NP}_{\mathbb{R}}}$$

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The oracle machines

An oracle: $\mathcal{O} \subseteq \mathbb{R}^\infty$.

The oracle machines:

$$\text{if } (Z_1, \ldots, Z_{I_1}) \in \mathcal{O} \text{ then goto } l_1 \text{ else goto } l_2.$$

$\Rightarrow$

$$\mathrm{P}_{\mathbb{R}} \subseteq \mathrm{DNP}_{\mathbb{R}} \subseteq \mathrm{NP}_{\mathbb{R}}.$$

$$\mathrm{P}^{\mathcal{O}}_{\mathbb{R}} \subseteq \mathrm{DNP}^{\mathcal{O}}_{\mathbb{R}} \subseteq \mathrm{NP}^{\mathcal{O}}_{\mathbb{R}}.$$

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A summary

Structure	$P \neq DNP$	$DNP \neq NP$	$P^Q \neq DNP^Q$	$DNP^Q \neq NP^Q$
$(\mathbb{Z};\mathbb{Z};\cdot,+,-;=)$	?	yes	defined analogously to BGS	$\emptyset$
$(\mathbb{Z};\mathbb{Z};\cdot,+,-;\geq)$	?	yes	defined analogously to BGS	$\emptyset$
$(\mathbb{R};\mathbb{R};\cdot,+,-;=)$	?	yes	derived from BGS or KP	$\emptyset$
$(\mathbb{R};\mathbb{R};\cdot,+,-;\geq)$	?	?	defined now	$\mathbb{Z}$, $\mathbb{Q}$, E

BGS: Baker-Gill-Solovay oracle, E: Emerson oracle, KP: Knapsack Problem

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cp. CCA 2008

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An oracle Q with $P_R^Q \neq DNP_R^Q$

Diagonalization techniques from Baker, Gill, and Solovay (only 0 and 1 as constants)

$i \in \mathbb{N}^+$	the code of a pair (p_i, P_i)
p_i	polynomial
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$$\mathcal{Q}_{\mathsf{R}} = \bigcup_{i \geq 1} W_i,$$

Important:

$V_i \neq \emptyset$ iff $\mathcal{N}_i^{W_i}$ rejects $(0,\ldots,0) \in \mathbb{R}^{n_i}$

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$V_i \neq \emptyset$ can be satisfied

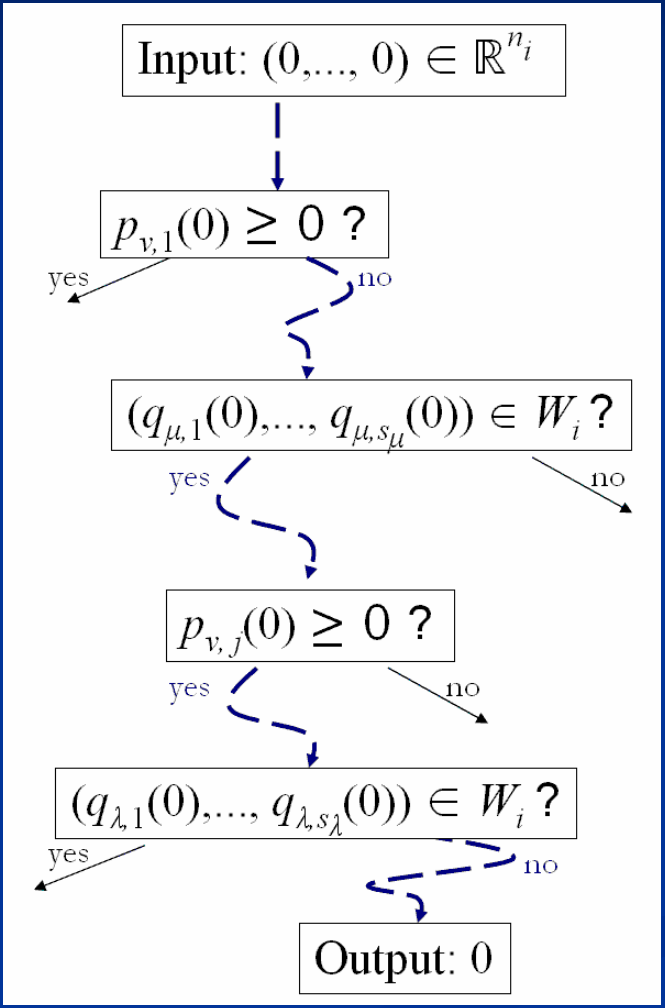
since
the path of $\mathcal{N}_i^{W_i}$ traversed by $(0,\ldots,0) \in \mathbb{R}^{n_i}$

- is uniquely determined
- of polynomial length

$\mathcal{N}_i^{W_i}$ **rejects** $(0,\ldots,0) \in \mathbb{R}^{n_i}$

Diagonalization techniques from Baker, Gill, and Solovay (only 0 and 1 as constants)

$\mathcal{N}_i^{W_i}$:



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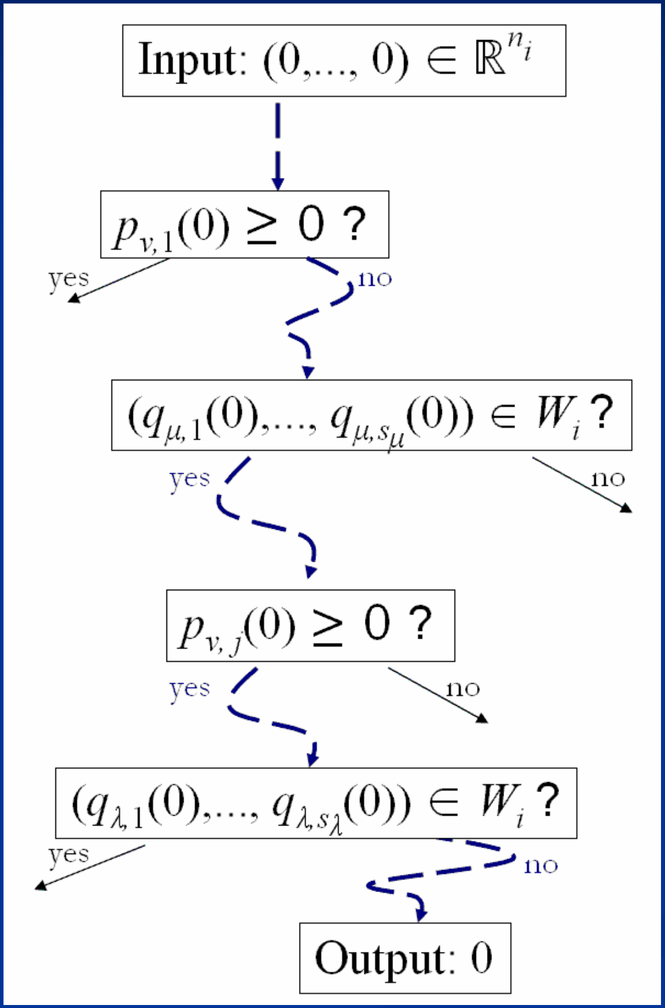

Only 0 and 1 as constants encoded by themselves

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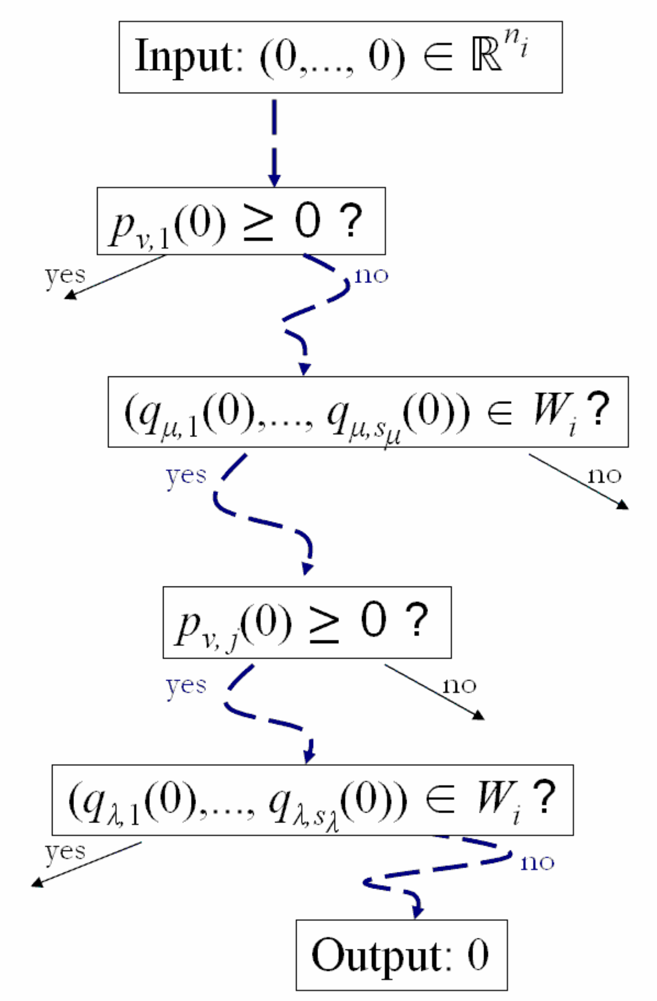

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$\Rightarrow$

The polynomials are uniquely determined.

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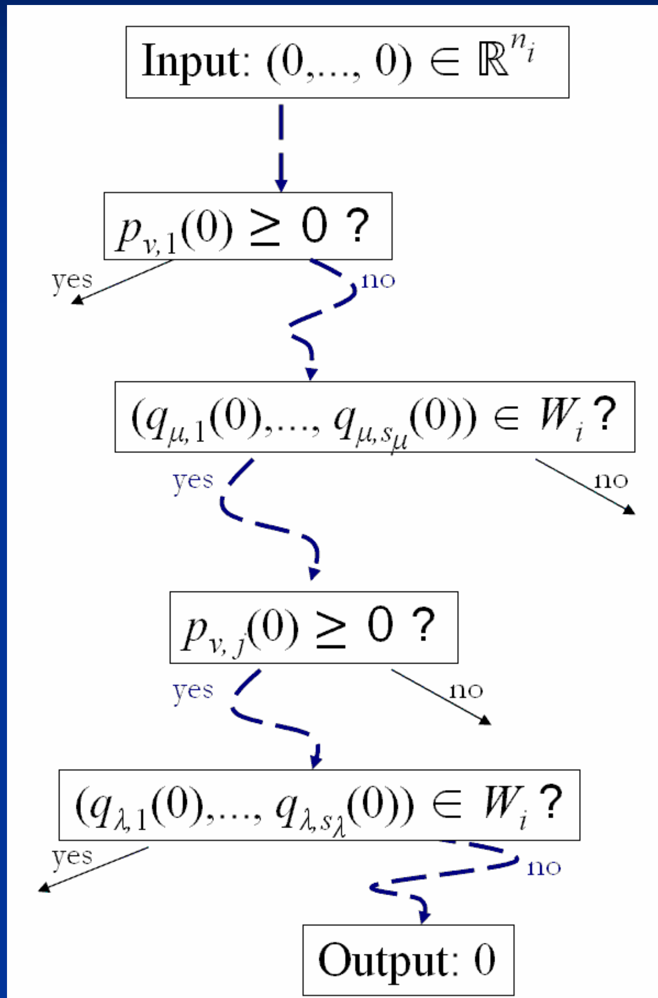

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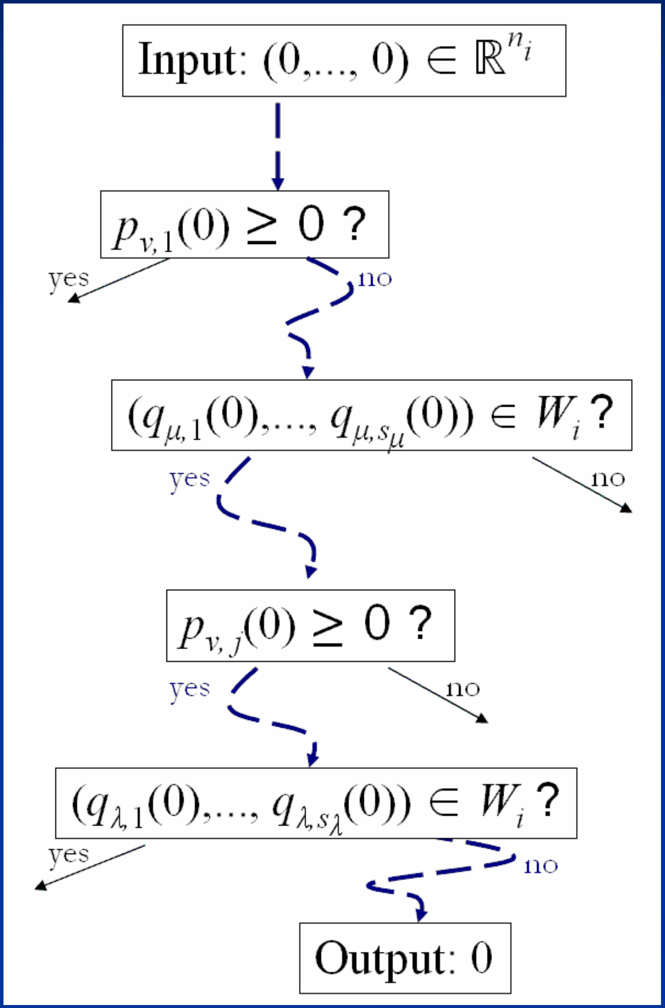

length $\leq 2^{n_i}$

$\Rightarrow \exists \boldsymbol{x} \in \{0, 1\}^{n_i}$

($\boldsymbol{x}$ is not queried by $\mathcal{N}_i^{W_i}$)

$\Rightarrow V_i \neq \emptyset$

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$$\mathcal{Q}_{\mathrm{R}} = \cup_{i \geq 1} W_i,$$

Important: $\mathcal{N}_i^{Q} \triangleq \mathcal{N}_i^{W_{i+1}} \triangleq \mathcal{N}_i^{W_i}$ on $(0,\ldots,0) \in \mathbb{R}^{n_i}$

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$m_i = 2^{n_i}$.

$\Rightarrow \mathcal{N}_i^{W_{i+1}} \triangleq \mathcal{N}_i^{Q}$ on $(0,\ldots,0) \in \mathbb{R}^{n_i}$

$\mathcal{Q}_{\mathrm{R}} = \bigcup_{i \geq 1} W_i$,

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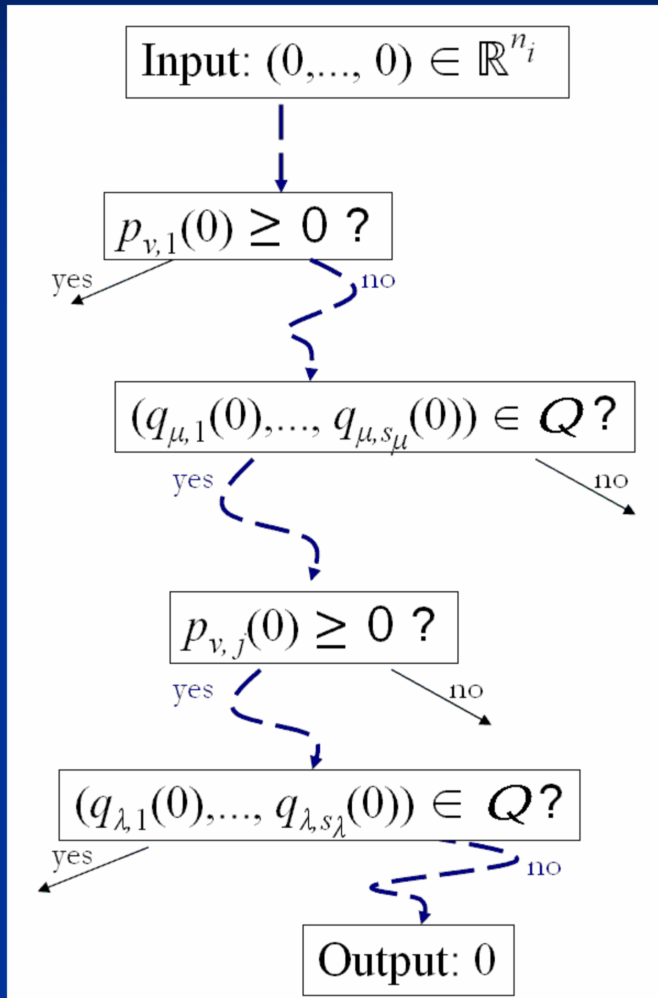

s_μ , $s_\lambda \leq n_{i+1}$

$\Downarrow$

$\triangleq$ $\ldots \in W_{i+1}$?

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$$m_i = 2^{n_i}.$$

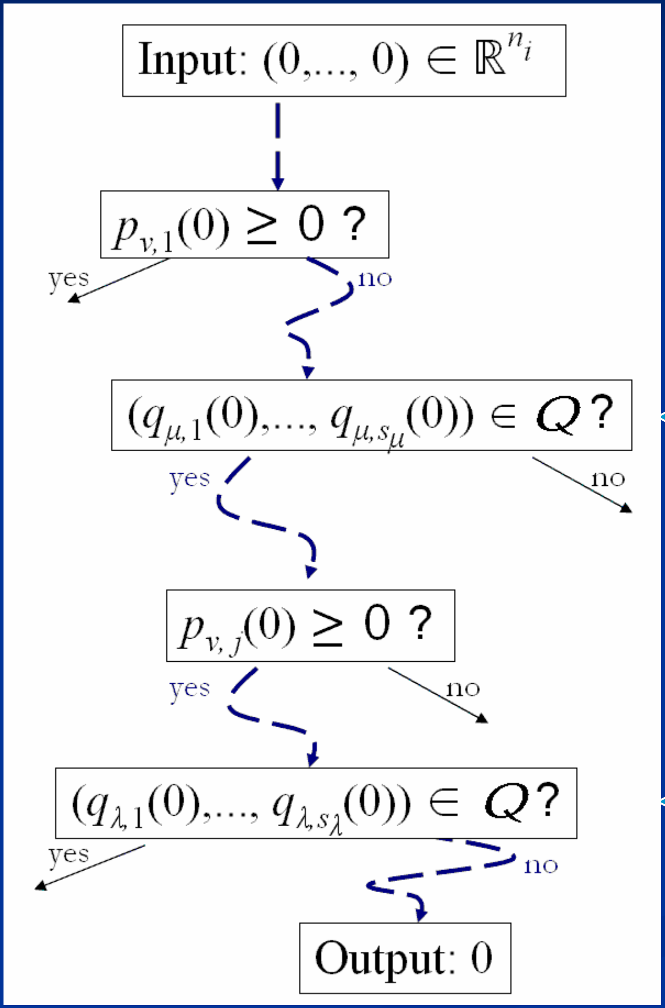
$\Rightarrow \mathcal{N}_i^{W_{i+1}} \triangleq \mathcal{N}_i^{W_i}$ on $(0,\dots,0) \in \mathbb{R}^{n_i}$

$$\mathcal{Q}_{\mathrm{R}} = \cup_{i \geq 1} W_i,$$

$\mathcal{N}_i^{W_i} \triangleq \mathcal{N}_i^{W_{i+1}} \triangleq \mathcal{N}_i^{Q}$ on $(0,\dots,0) \in \mathbb{R}^{n_i}$

Diagonalization techniques from Baker, Gill, and Solovay (only 0 and 1 as constants)

$\mathcal{N}_i^{Q}$:



$x \in V_i \Rightarrow x \in W_{i+1}$

$\Rightarrow$ x is not queried by $\mathcal{N}_i^{W_i}$

$\Downarrow$

$\triangleq \quad \dots \in W_i$?

$\triangleq \quad \dots \in W_i$?

An oracle Q with $P_R^Q \neq DNP_R^Q$

Diagonalization techniques from Baker, Gill, and Solovay (only 0 and 1 as constants)

BSS - only with 0 and 1

$i \in \mathbb{N}^+$	the code of a pair (p_i, P_i)
p_i	polynomial
P_i	program of a $\mathrm{P}^{\mathcal{O}}$-machine using only the constants 0 and 1
$\mathcal{N}_i^{\mathcal{B}}$	machine using $\mathcal{B} \subseteq \mathbb{R}^\infty$, P_i, and the time bound p_i

$V_0 = \emptyset$, $m_0 = 0$. Stage $i \geq 1$: Let $n_i > m_{i-1}$ and $p_i(n_i) + n_i < 2^{n_i}$.

$$W_i = \cup_{j<i} V_j,$$

$$V_i = \{\mathbf{x} \in \{0,1\}^{n_i} \mid \mathcal{N}_i^{W_i} \text{ rejects } (0,\ldots,0) \in \mathbb{R}^{n_i}$$
$$\& \ \mathbf{x} \text{ is not queried by } \mathcal{N}_i^{W_i} \text{ on input } (0,\ldots,0) \in \mathbb{R}^{n_i}\},$$

$$m_i = 2^{n_i}.$$

$$\mathcal{Q}_\mathsf{R} = \cup_{i \geq 1} W_i,$$

$$L_\mathsf{R} = \{\mathbf{y} \mid (\exists i \in \mathbb{N}^+)(\mathbf{y} \in \mathbb{R}^{n_i} \ \& \ V_i \neq \emptyset)\} \in \mathrm{DNP}_\mathsf{R}^{\mathcal{Q}_\mathsf{R}} \setminus \mathrm{P}_\mathsf{R}^{\mathcal{Q}_\mathsf{R}}.$$

$$\mathsf{R} = (\mathbb{R}; 0, 1; \cdot, +, -; \geq).$$

An oracle Q with $P_R^{\,Q} \neq DNP_R^{\,Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only 0 and 1 as constants)

BSS - only with 0 and 1

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An oracle Q with $\mathrm{P}_{\mathbb{R}_{(=)}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}_{(=)}}^{Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

$i \in \mathbb{N}^+$	the code of a pair (p_i, P_i)
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$1, \ldots, k_i$	codes of $c_1, \ldots, c_{k_i} \in \mathbb{R}$
$\mathcal{N}_i^{\mathcal{B}, c_1, \ldots, c_{k_i}}$	uses $\mathcal{B} \subseteq \mathbb{R}^\infty$, $c_1, \ldots, c_{k_i}$, P_i, and the time bound p_i

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$$V_i = \{\mathbf{x} \in \{0,1\}^{n_i} \mid \forall c_1 \cdots \forall c_{k_i} (\mathcal{N}_i^{W_i, c_1, \ldots, c_{k_i}} \text{ rejects } (0, \ldots, 0) \in \mathbb{R}^{n_i} \;\&\; \mathbf{x} \text{ is not queried by } \mathcal{N}_i^{W_i, c_1, \ldots, c_{k_i}} \text{ on } (0, \ldots, 0) \in \mathbb{R}^{n_i})\}$$

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An oracle Q with $\mathrm{P}_{\mathbb{R}_{(=)}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}_{(=)}}^{Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

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$V_i = \emptyset$ can be satisfied although x is not queried on (0,...,0)

An oracle Q with $\mathrm{P}_{\mathbb{R}_{(=)}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}_{(=)}}^{Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

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↯

An oracle Q with $\mathrm{P}_{\mathbb{R}(=)}^{Q} \neq \mathrm{DNP}_{\mathbb{R}(=)}^{Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

only equality tests

$\sum_{j_1,\ldots,j_k \geq 0} \alpha_{j_1,\ldots,j_k} c_1^{j_1} \cdots c_k^{j_k} \in \{0,1\}$?

where $k \leq k_i$, $c_1, \ldots, c_{k_i} \in \mathbb{R}$, $\alpha_{j_1,\ldots,j_k} \in \mathbb{Z}$.

$$\begin{array}{l} p(c_k) = 0? \\ p(c_k) = 1? \end{array} \quad \text{for} \quad p(x) = \sum_{j_1,\ldots,j_k \geq 0} \alpha_{j_1,\ldots,j_k} c_1^{j_1} \cdots c_{k-1}^{j_{k-1}} x^{j_k}.$$

$F_0 = \mathbb{Q}$. Stage $j = 1, \ldots, k_i$:

$$\begin{array}{lll} F_j = F_{j-1}, & d_j = 1 & \text{if } c_j \in F_{j-1}, \\ F_j = F_{j-1}(c_j), & d_j = \infty & \text{if } c_j \text{ is not algebraic over } F_{j-1}, \\ F_j = F_{j-1}[c_j], & d_j = m & \text{if there is an irreducible } q_j \in F_{j-1}[x], \\ & & \text{degree}(q_j) \geq 2 \ \& \ q_j(c_j) = 0. \end{array}$$

$$\begin{array}{lll} d_k = 1 & \Rightarrow & c_k \text{ is not necessary.} \\ d_k = \infty & \Rightarrow & p(c_k) \neq 0. \\ d_k \geq 2 & \Rightarrow & p(c_k) = 0 \text{ iff } q_k | p. \end{array}$$

cp. CCC 2009

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An oracle Q with $\mathrm{P}_{\mathbb{R}_{(=)}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}_{(=)}}^{Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

only equality tests

The answer to $p(c_k) = 0$? and $p(c_k) = 1$? is only dependent on some

$$\mathrm{char}(c_1, \ldots, c_{k_i}) = (d_1, \ldots, d_{k_i}, q_1, \ldots, q_{k_i})$$

where $d_k \geq 2 \Rightarrow q_k(c_k) = 0$ and q_k irreducible.

$i \in \mathbb{N}^+$ the code of (p_i, P_i, t_i),

$t_i \ = (d_1, \ldots, d_{k_i}, q_1, \ldots, q_{k_i})$,

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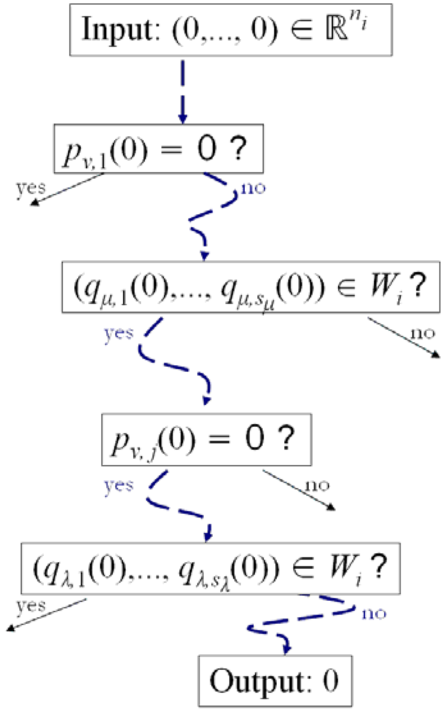
The path of $\mathcal{N}_i^{W_i, c_1, \ldots, c_{k_i}}$ is uniquely determined.



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Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

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An oracle Q with $P_{\mathbb{R}_{(=)}}^{Q} \neq DNP_{\mathbb{R}_{(=)}}^{Q}$

Diagonalization techniques from Baker, Gill, and Solovay (only equality tests)

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$$\begin{aligned} V_i = \{\mathbf{x} \in \{0,1\}^{n_i} \mid \forall c_1 \cdots \forall c_{k_i} (&\mathrm{char}(c_1, \ldots, c_{k_i}) = t_i \\ &\& \ \mathcal{N}_i^{W_i, c_1, \ldots, c_{k_i}} \text{ rejects } (0, \ldots, 0) \in \mathbb{R}^{n_i} \\ &\& \ \mathbf{x} \text{ is not queried by } \mathcal{N}_i^{W_i, c_1, \ldots, c_{k_i}} \text{ on } (0, \ldots, 0) \in \mathbb{R}^{n_i})\}. \end{aligned}$$

$$L_{\mathbb{R}_{(=)}} = \{\mathbf{y} \mid (\exists i \in \mathbb{N}^+)(\mathbf{y} \in \mathbb{R}^{n_i} \ \& \ V_i \neq \emptyset)\} \in \mathrm{DNP}_{\mathbb{R}_{(=)}}^{\mathcal{Q}_{\mathbb{R}_{(=)}}} \setminus \mathrm{P}_{\mathbb{R}_{(=)}}^{\mathcal{Q}_{\mathbb{R}_{(=)}}}.$$

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An oracle Q with $P_{\mathbb{R}}^{Q} \neq DNP_{\mathbb{R}}^{Q}$

Problems in the full BSS model

BSS - with order tests

$i \in \mathbb{N}^+$	the code of a pair (p_i, P_i)
p_i	polynomial
P_i	program of a $\mathrm{P}^{\mathcal{O}}$-machine containing $1, \ldots, k_i$ for $c_1, \ldots, c_{k_i}$
$\mathcal{K}_{i,j}^{\mathcal{B}}$	set of machines using $\mathcal{B}$, $c_1, \ldots, c_{k_i}$, P_i, p_i, described by $N_{i,j}$

An oracle Q with $\mathrm{P}_{\mathbb{R}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}}^{Q}$

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?

An oracle Q with $\mathrm{P}_{\mathbb{R}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}}^{Q}$

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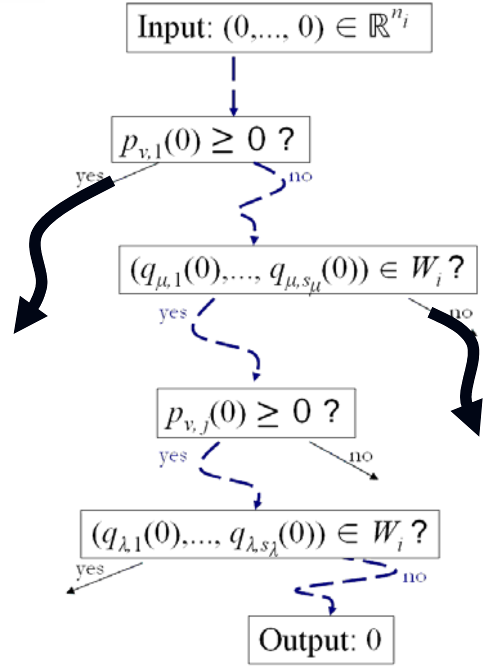
!!!



Characterization of the algebraic dependence of the constants $c_1,\ldots,c_{k_i}$ is **not** sufficient

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An oracle Q with $\mathrm{P}_{\mathbb{R}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}}^{Q}$

Problems in the full BSS model

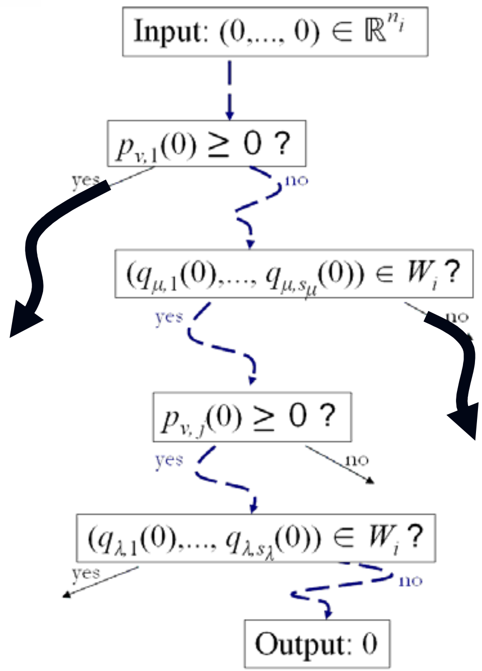
BSS - with order tests

$i \in \mathbb{N}^+$ the code of a pair (p_i, P_i)

p_i polynomial

P_i program of a $\mathrm{P}^{\mathcal{O}}$-machine containing $1, \ldots, k_i$ for $c_1, \ldots, c_{k_i}$

$\mathcal{K}^{\mathcal{B}}_{i,j}$ set of machines using $\mathcal{B}$, $c_1, \ldots, c_{k_i}$, P_i, p_i, described by $N_{i,j}$



$\Rightarrow$ We need a new encoding.

But: If n_i will be greater, then the test results are also dependent on the new zeros of the new polynonials.

An oracle Q with $P_{\mathbb{R}}^{Q} \neq DNP_{\mathbb{R}}^{Q}$

Problems in the full BSS model

BSS - with order tests

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$\mathcal{K}_{i,j}^{\mathcal{B}}$	set of machines using $\mathcal{B}$, $c_1, \ldots, c_{k_i}$, P_i, p_i, described by $N_{i,j}$

$m_0 = 0.$

Stage $i \geq 1$: $n_i > m_{i-1}$, $p_i(n_i) < 2^{n_i - 1}$, $p_i(n_i) + n_i < 2^{n_i}$. $V_{i,0} = \emptyset$.

Stage $j \geq 1$:

$$W_{i,j} = \cup_{i'<i} V_{i'} \cup \cup_{j'<j} V_{i,j'},$$

$$V_{i,j} = \{\mathbf{x} \in \{0,1\}^{n_i - 1} \times \{C_{i,j}\} \mid$$

?

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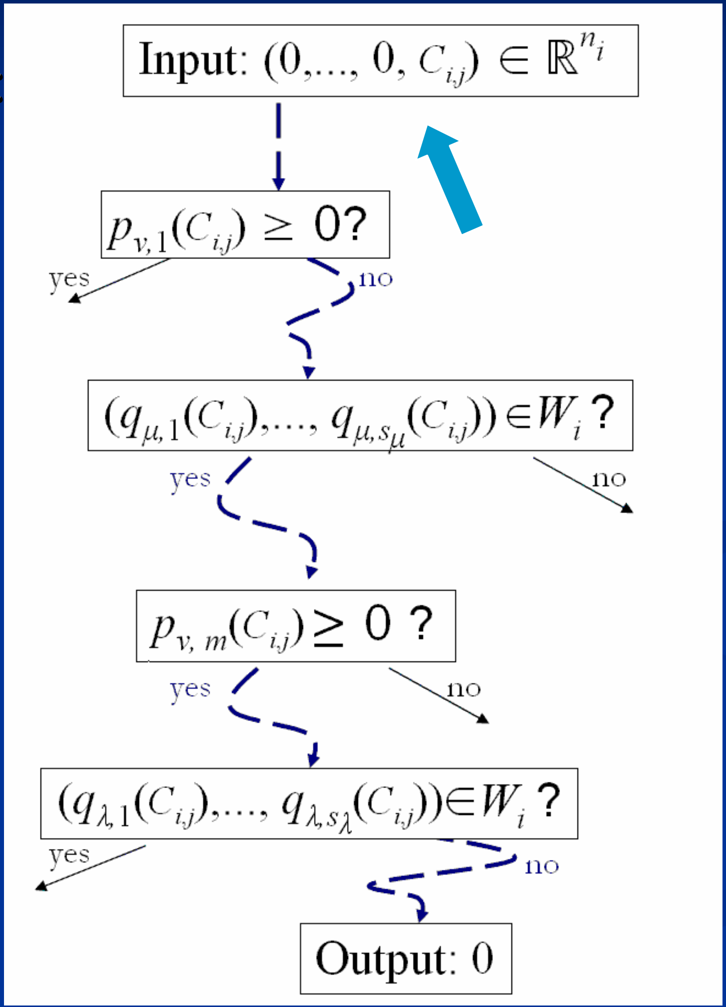
$\mathcal{N}_i^{W_i,c_1,\dots,c_{k_i}}$ rejects $(0,\dots,0,C_{i,j}) \in \mathbb{R}^{n_i}$?

Using further ideas for the full BSS model

$\mathcal{N}_i^{W_i,c_1,\dots,c_{k_i}} \in K_{i,j}$:



The values of the polynomials at $C_{i,j}$ are uniquely determined by $C_{i,j}$.

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$\mathcal{N}_i^{W_i,c_1,\dots,c_{k_i}}$ rejects $(0,\dots,0,C_{i,j}) \in \mathbb{R}^{n_i}$?

Using further ideas for the full BSS model

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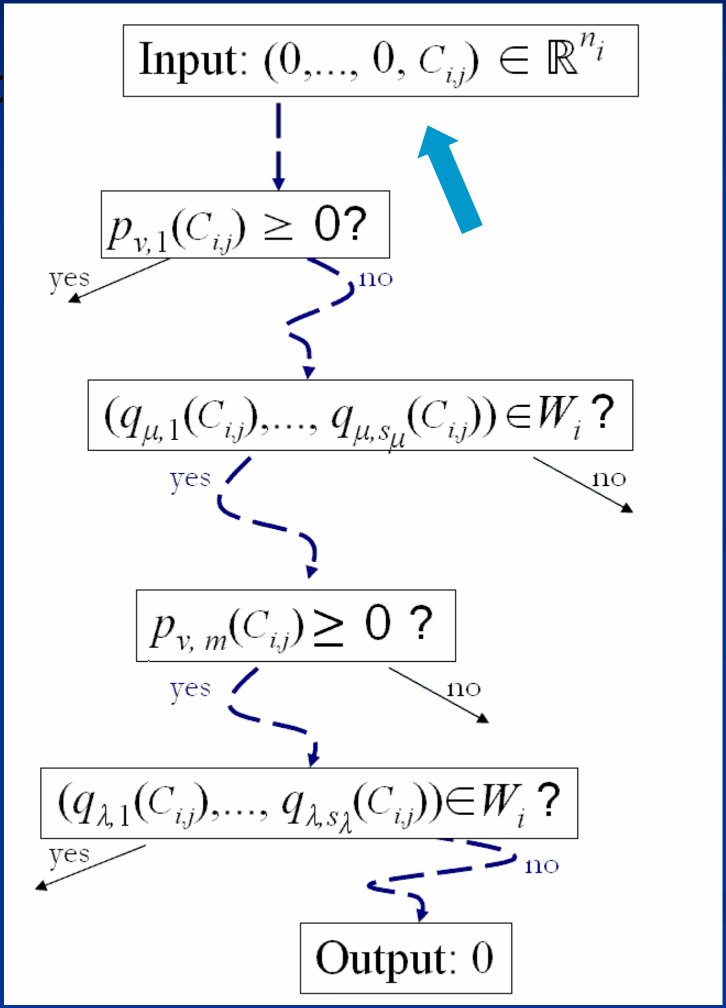

The values of the polynomials at $C_{i,j}$ are uniquely determined by $C_{i,j}$.

$\Rightarrow$

The path is uniquely determined.

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$
$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \left(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \alpha^{(k)}_{j_1,\ldots,j_{k_i},j}\, c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\right) x^j.$$

The definition of $C_{i,j}$

BSS - with order tests

An enumeration

$f_1, f_2, \ldots, f_s$

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \left(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \alpha^{(k)}_{j_1,\ldots,j_{k_i},j}\, c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\right) x^j.$$

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

All possible coefficients are considered.

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

$\mu =$

$\mu' =$

$\nu =$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$
$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\text{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k = \text{code}(f_k) \in \mathbb{N}^+$ if $f_k \in \mathbb{Q}[x]$,

$\mu_k =$

$\nu_k =$

$\mu =$

$\mu' =$

$\nu =$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k = \mathrm{code}(f_k) \in \mathbb{N}^+$ if $f_k \in \mathbb{Q}[x]$,

$\mu_k =$

$\nu_k =$

$\mu =$

$\mu' =$

$\nu =$

determines order tests $p_{\nu,\mu}(x) \geq 0$ and queries on $(0,\ldots,0, N)$ if $N \in \mathbb{N}$, $p_{\nu,\mu}\, q_{\lambda,\mu} \in \mathbb{Q}[x]$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\text{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k = \text{code}(f_k) \in \mathbb{N}^+$ if $f_k \in \mathbb{Q}[x]$,

$\mu_k = 0$ otherwise,

$\nu_k =$

$\mu =$

$\mu' =$

$\nu =$

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\text{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

$\mu \ = \min \bigcap_{\substack{k=1,\ldots,s \\ \text{degree}(f_k)>1}} \{n \in \mathbb{N} \mid \forall x (f_k(x) = 0 \vee f_k(x) = 1 \Rightarrow n > x)\},$

$\mu' =$

$\nu \ =$

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

$\mu = \min \bigcap_{\substack{k=1,\ldots,s \\ \mathrm{degree}(f_k)>1}} \{n \in \mathbb{N} \mid \forall x (f_k(x) = 0 \vee f_k(x) = 1 \Rightarrow n > x)\},$

greater than the zeros

$\mu' =$

$\nu =$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k = \lim_{x\to\infty} \mathrm{sgn}(f_k(x)),$

$\mu =$

$\mu' =$

$\nu =$

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}} \Big) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$$\mu_k =$$

$$\mu_k =$$

$$\nu_k = \lim_{x\to\infty} \mathrm{sgn}(f_k(x)),$$

$$\mu =$$

$$\mu' =$$

$$\nu =$$

determine order tests $p_{\nu,\mu}(x) \geq 0$
on $(0, \ldots, 0, N)$
for large $N \in \mathbb{N}$

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

determine order tests $p_{\nu,\mu}(x) \geq 0$
on $(0,\ldots, 0, N)$
for large $N \geq N_{\mathrm{char}}(\ldots)$

$$\mu_k =$$
$$\mu_k =$$
$$\nu_k = \lim_{x\to\infty} \mathrm{sgn}(f_k(x)),$$
$$\mu = \min \bigcap_{\substack{k=1,\ldots,s \\ \mathrm{degree}(f_k)>1}} \{n \in \mathbb{N} \mid \forall x (f_k(x) = 0 \vee f_k(x) = 1 \Rightarrow n > x)\},$$
$$\mu' =$$
$$\nu =$$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$$\mu_k =$$
$$\mu_k =$$
$$\nu_k =$$
$$\mu =$$
$$\mu' = \min \bigcap_{\substack{k=1,\ldots,s \\ \mu_k=0}} \{n \in \mathbb{N} \mid (\forall x \in \mathbb{N})(f_k(x) \in \mathbb{N} \Rightarrow n > x)\},$$
$$\nu =$$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

$\mu =$

$\mu' = \min \bigcap_{\substack{k=1,\ldots,s \\ \mu_k=0}} \{n \in \mathbb{N} \mid (\forall x \in \mathbb{N})(f_k(x) \in \mathbb{N} \Rightarrow n > x)\},$

$\nu =$

greater than all natural values

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

determine parts of the queries on $(0,\ldots, 0, N)$ for large $N \geq N_{\mathrm{char}}(\ldots)$, $N \in \mathbb{N}$

$$\mu = \min \bigcap_{\substack{k=1,\ldots,s \\ \mathrm{degree}(f_k)\geq 1}} \{n \in \mathbb{N} \mid \forall x (f_k(x) = 0 \vee f_k(x) = 1 \Rightarrow n > x)\},$$

$$\mu' = \min \bigcap_{\substack{k=1,\ldots,s \\ \mu_k=0}} \{n \in \mathbb{N} \mid (\forall x \in \mathbb{N})(f_k(x) \in \mathbb{N} \Rightarrow n > x)\},$$

$\nu =$

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The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

$\mu =$

$\mu' =$

$\nu = \min \bigcap_{k=1,\ldots,s} \{n \in \mathbb{N} \mid f_k(n) < 2^n\}.$

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \sum_{j=0}^{2^{p_i(n_i)}} \Big(\sum_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)},\, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}\Big) x^j.$$

$N_{\mathrm{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$\mu_k =$

$\mu_k =$

$\nu_k =$

$\mu =$

$\mu' =$

$\nu = \min \bigcap_{k=1,\ldots,s} \{ n \in \mathbb{N} \mid f_k(n) < 2^n \}.$

allows to extend the oracle step-by-step by a recursive definition

The definition of $C_{i,j}$

BSS - with order tests

$f_1, f_2, \ldots, f_s$ where

$$f_k \in \mathbb{R}[x],$$

$$f_k(x) = \Sigma_{j=0}^{2^{p_i(n_i)}} (\Sigma_{j_1,\ldots,j_{k_i}=0}^{2^{p_i(n_i)}} \underbrace{\alpha^{(k)}_{j_1,\ldots,j_{k_i},j}}_{\in \mathbb{Z} \cap [-2^{p_i(n_i)}, 2^{p_i(n_i)}]} c_1^{j_1} \cdots c_{k_i}^{j_{k_i}}) x^j.$$

$N_{\text{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$ where

$$\begin{aligned}
\mu_k &= \text{code}(f_k) \in \mathbb{N}^+ && \text{if } f_k \in \mathbb{Q}[x], \\
\mu_k &= 0 && \text{otherwise}, \\
\nu_k &= \lim_{x \to \infty} \text{sgn}(f_k(x)), \\
\mu &= \min \bigcap_{\substack{k=1,\ldots,s \\ \text{degree}(f_k) \geq 1}} \{n \in \mathbb{N} \mid \forall x (f_k(x) = 0 \vee f_k(x) = 1 \Rightarrow n > x)\}, \\
\mu' &= \min \bigcap_{\substack{k=1,\ldots,s \\ \mu_k = 0}} \{n \in \mathbb{N} \mid (\forall x \in \mathbb{N})(f_k(x) \in \mathbb{N} \Rightarrow n > x)\}, \\
\nu &= \min \bigcap_{k=1,\ldots,s} \{n \in \mathbb{N} \mid f_k(n) < 2^n\}.
\end{aligned}$$

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The definition of $C_{i,j}$

BSS - with order tests

$N_{\text{char}}(i, c_1, \ldots, c_{k_i})$

Cantor number of $(\mu_1, \ldots, \mu_s, \nu_1, \ldots, \nu_s, \mu, \mu', \nu)$

$N_{i,1}, N_{i,2}, \ldots$

an enumeration of

$$\{N_{\text{char}}(i, c_1, \ldots, c_{k_i}) \mid c_1, \ldots, c_{k_i} \in \mathbb{R}\},$$

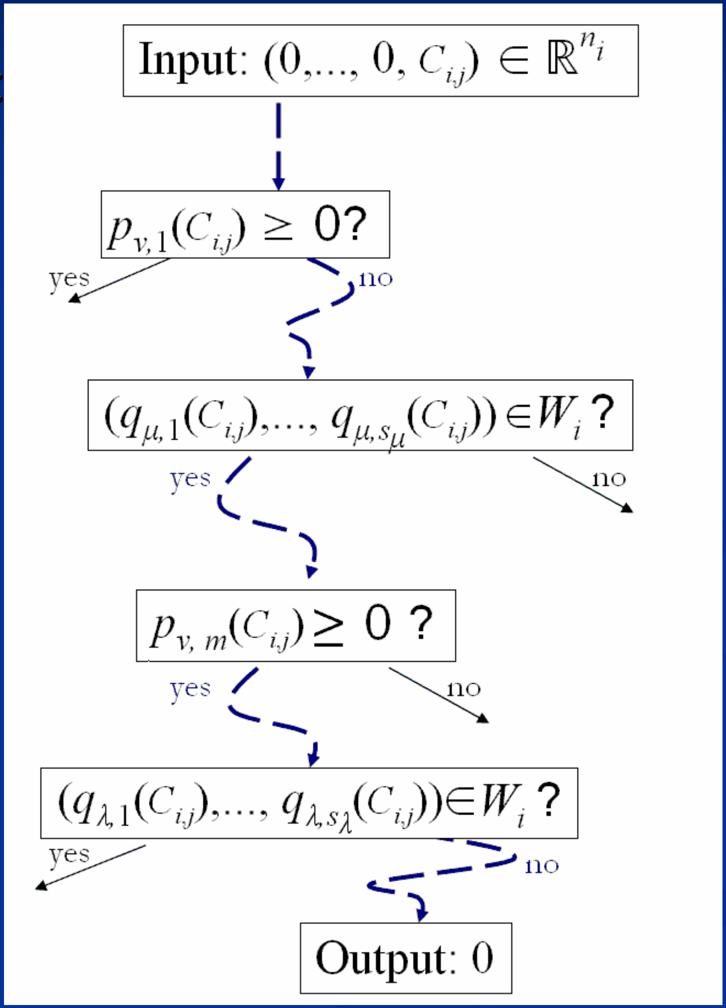
$N_{i,j+1} > N_{i,j}$.

$$C_{i,j} = \max\{2^{C_{i,j-1}}, N_{i,j}\}.$$

$$\mathcal{K}_{i,j}^{\mathcal{B}} = \{\mathcal{N}_i^{\mathcal{B}, c_1, \ldots, c_{k_i}} \mid N_{i,j} = N_{\text{char}}(i, c_1, \ldots, c_{k_i})\} \ .$$

$\mathcal{N}_i^{W_i,c_1,\ldots,c_{k_i}}$ rejects $(0,\ldots,0,C_{i,j}) \in \mathbb{R}^{n_i}$

$\mathcal{N}_i^{W_i,c_1,\ldots,c_{k_i}} \in K_{i,j}$:



The values of the polynomials at $C_{i,j}$ are uniquely determined by $C_{i,j}$.

$\Rightarrow$

The path is uniquely determined.

An oracle Q with $P_{\mathbb{R}}^{Q} \neq DNP_{\mathbb{R}}^{Q}$

Problems in the full BSS model

BSS - with order tests

$i \in \mathbb{N}^+$ the code of a pair (p_i, P_i)
p_i polynomial
P_i program of a $\mathrm{P}^{\mathcal{O}}$-machine containing $1, \ldots, k_i$ for $c_1, \ldots, c_{k_i}$
$\mathcal{K}^{\mathcal{B}}_{i,j}$ set of machines using $\mathcal{B}$, $c_1, \ldots, c_{k_i}$, P_i, p_i, described … $N_{i,j}$

$m_0 = 0.$

Stage $i \geq 1$: $n_i > m_{i-1}$, $p_i(n_i) < 2^{n_i - 1}$, $p_i($ … $\emptyset$.

See my paper.

Stage $j \geq 1$:

$$W_{i,j} = \cup_{i'<i} V_{i'} \cup \cup_{j'<j} V_{i,j'},$$
$$V_{i,j} = \{\mathbf{x} \in \{0,1\}^{n_i - 1} \times \{C_{i,j}\} \mid (\exists \mathcal{M} \in \mathcal{K}^{W_{i,j}}_{i,j})(\mathcal{M} \text{ rejects } (0, \ldots, 0, C_{i,j}) \;\&\; \mathbf{x} \text{ is not queried by } \mathcal{M} \text{ on input } (0, \ldots, 0, C_{i,j}) \in \mathbb{N}^{n_i})\}.$$

$m_i = 2^{n_i}$, $V_i = \cup_{j \geq 1} V_{i,j}$.

$\mathcal{Q}_1 = \cup_{i \geq 1} V_i$,
$L_1 = \cup_{i \geq 1} \{(y_1, \ldots, y_{n_i - 1}, N) \in \mathbb{R}^{n_i} \mid V_i \cap (\{0,1\}^{n_i - 1} \times \{N\}) \neq \emptyset)\}$.
$\Rightarrow L_1 \in \mathrm{DNP}^{\mathcal{Q}_1}_{\mathbb{R}} \setminus \mathrm{P}^{\mathcal{Q}_1}_{\mathbb{R}}$.

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A second oracle Q with $\mathrm{P}_{\mathbb{R}}^{Q} \neq \mathrm{DNP}_{\mathbb{R}}^{Q}$

Problems in the full BSS model

$E_0 = \mathbb{Q}$, $\tau_1, \tau_2, \ldots$ where τ_{i+1} is transcendental over $E_i =_{\mathrm{df}} E_{i-1}(\tau_i)$

$A_n = \{(v_1, \ldots, v_{2n}) \in \{0, v\}^{2n} \mid v \in \mathbb{Z} \setminus \{0\} \ \& \ \sum_{i=1}^{2n} v_i = nv\}.$

$\mathcal{Q}_2 = \bigcup_{n=1}^{\infty} \{(\mathrm{sgn}(|v_1|), \ldots, \mathrm{sgn}(|v_{2n}|), \sum_{i=1}^{2n} v_i\tau_i) \in \mathbb{R}^{2n+1} \mid (v_1, \ldots, v_{2n}) \in A_n\}.$

$L_2 = \bigcup_{n=1}^{\infty} \{(0, \ldots, 0, \sum_{i=1}^{2n} v_i\tau_i) \in \mathbb{R}^{2n+1} \mid (v_1, \ldots, v_{2n}) \in A_n\}.$

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A second oracle $\mathcal{Q}$ with $\mathrm{P}_{\mathbb{R}}^{\mathcal{Q}} \neq \mathrm{DNP}_{\mathbb{R}}^{\mathcal{Q}}$

Problems in the full BSS model

$E_0 = \mathbb{Q}$, $\tau_1, \tau_2, \ldots$ where τ_{i+1} is transcendental over $E_i =_{\mathrm{df}} E_{i-1}(\tau_i)$

$A_n = \{(v_1, \ldots, v_{2n}) \in \{0, v\}^{2n} \mid v \in \mathbb{Z} \setminus \{0\} \ \& \ \sum_{i=1}^{2n} v_i = nv\}$.

$\mathcal{Q}_2 = \cup_{n=1}^{\infty}\{(\mathrm{sgn}(|v_1|), \ldots, \mathrm{sgn}(|v_{2n}|), \sum_{i=1}^{2n} v_i\tau_i) \in \mathbb{R}^{2n+1} \mid (v_1, \ldots, v_{2n}) \in A_n\}$.

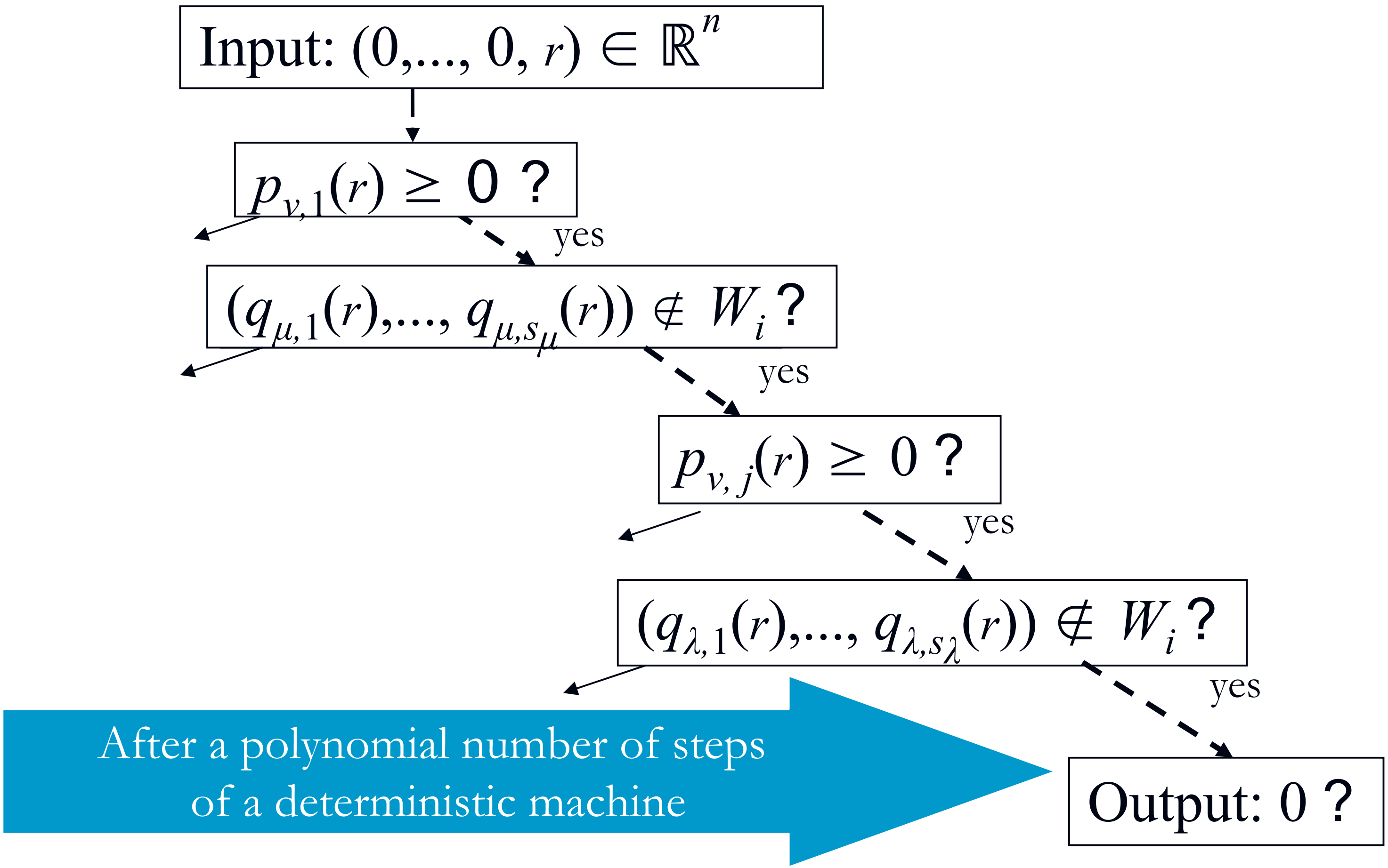
$L_2 = \cup_{n=1}^{\infty}\{(0, \ldots, 0, \sum_{i=1}^{2n} v_i\tau_i) \in \mathbb{R}^{2n+1} \mid (v_1, \ldots, v_{2n}) \in A_n\}$.

$\Rightarrow$ Each computation path of a $\mathrm{P}^{\mathcal{Q}_2}$-machine is traversed by $(0, \ldots, 0, x)$ only if x satisfies some

$$(z_1, \ldots, z_s, p_k(x)) \notin \mathcal{Q}_2, \qquad (z_1, \ldots, z_s, p_k(x)) \in \mathcal{Q}_2.$$

A second oracle Q with $P_{\mathbb{R}}^{Q} \neq DNP_{\mathbb{R}}^{Q}$

Problems in the full BSS model



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A second oracle Q with $P_{\mathbb{R}}^{Q} \neq DNP_{\mathbb{R}}^{Q}$

Problems in the full BSS model

$E_0 = \mathbb{Q}$, $\tau_1, \tau_2, \ldots$ where τ_{i+1} is transcendental over $E_i =_{\mathrm{df}} E_{i-1}(\tau_i)$

$$A_n = \{(v_1, \ldots, v_{2n}) \in \{0, v\}^{2n} \mid v \in \mathbb{Z} \setminus \{0\} \ \& \ \Sigma_{i=1}^{2n} v_i = nv\}.$$

$$\mathcal{Q}_2 = \cup_{n=1}^{\infty} \{(\mathrm{sgn}(|v_1|), \ldots, \mathrm{sgn}(|v_{2n}|), \Sigma_{i=1}^{2n} v_i \tau_i) \in \mathbb{R}^{2n+1} \mid (v_1, \ldots, v_{2n}) \in A_n\}.$$

$$L_2 = \cup_{n=1}^{\infty} \{(0, \ldots, 0, \Sigma_{i=1}^{2n} v_i \tau_i) \in \mathbb{R}^{2n+1} \mid (v_1, \ldots, v_{2n}) \in A_n\}$$

$\Rightarrow$ Each computation path of a $P^{\mathcal{Q}_2}$-machine is t[illegible]$, 0, x)$
only if x satisfies some

$$(z_1, \ldots, z_s, p_k(x)) \notin \mathcal{Q}_2, \qquad (z_1, \ldots, z_s, p_k(x)) \ [illegible] \ \mathcal{Q}_2.$$

See my paper.

For any $P^{\mathcal{Q}_2}$-machine there is an i_0 such that

(1) $x = \Sigma_{i=i_0+1}^{2n} v_i \tau_i$,

(2) $v_l \neq 0$, $v_{l+1} = \cdots = v_n = 0$ $(i_0 < l \leq 2n)$,

(3) $(z_1, \ldots, z_s, p_k(x)) \in \mathcal{Q}_2$

$\Rightarrow s \geq 2n$ and $(z_{i_0+1}, \ldots, z_s) = (\mathrm{sgn}(|v_{i_0+1}|), \ldots, \mathrm{sgn}(|v_l|), 0, \ldots, 0)$.

$\Rightarrow L_2 \in DNP_{\mathbb{R}}^{\mathcal{Q}_2} \setminus P_{\mathbb{R}}^{\mathcal{Q}_2}$.

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A summary

Structure	$P \neq DNP$	$DNP \neq NP$	$P^{\mathcal{Q}} \neq DNP^{\mathcal{Q}}$	$DNP^{\mathcal{Q}} \neq NP^{\mathcal{Q}}$
$(\mathbb{Z}; \mathbb{Z}; \cdot, +, -; =)$	?	yes	defined analogously to BGS	$\emptyset$
$(\mathbb{Z}; \mathbb{Z}; \cdot, +, -; \geq)$	?	yes	defined analogously to BGS	$\emptyset$
$(\mathbb{R}; \mathbb{R}; \cdot, +, -; =)$	?	yes	derived from BGS or KP	$\emptyset$
$(\mathbb{R}; \mathbb{R}; \cdot, +, -; \geq)$	?	?	$\mathcal{Q}_1, \mathcal{Q}_2$	$\mathbb{Z}$, $\mathbb{Q}$, E

BGS: Baker-Gill-Solovay oracle, E: Emerson oracle, KP: Knapsack Problem

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Relativizations of the P =? DNP Question for the BSS Model

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