An Isomorphism Theorem for Partial Numberings

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As has been shown by the author, standard numberings of the computable real numbers and similar effectively given topological spaces are only partially defined, by necessity. Thus, not every natural number is a name of some computable object. It was demonstrated that any two such numberings are m-equivalent. Spaces like the partial computable functions, on the other hand, are known to have totally defined standard numberings such that any two of them are even recursively isomorphic.

In this talk it is studied whether such a result is also true for standard numberings of the computable reals and similar spaces. The investigation is carried out in the general setting of effective topological spaces introduced by the author. For total numberings it is well known that m-equivalent numberings are recursively isomorphic if they are precomplete. The proof proceeds in two steps: First it is shown that m-equivalent precomplete numberings are already 1-equivalent and then a generalization of Myhills theorem is applied. If one extends the usual reducibility relation between numberings to partial numberings in a straightforward way, the reduction function is allowed to map non-names with respect to one numbering onto names with respect to the other. A recursive isomorphism, however, can only map non-names onto non-names. If one allows only reduction functions operating in the same way – we speak of strong reducibility in this case –, the usual construction for Myhills theorem goes through and one obtains a generalization of this theorem to partial numberings.

In the second part of the talk the notion of admissible numbering of an effective space is strengthened in a similar way as was the reducibility notion. For the strongly admissible numberings thus obtained one has that any two of them are even strongly m-equivalent. A necessary and sufficient condition is presented for when such numberings are precompete.

As is finally shown, effective spaces have strongly admissible precomplete numberings. By the above results any two of them are recursively isomorphic.