Continuity in Constructive Mathematics

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As (hopefully) every student of mathematics learns in their first semester there are various notions of continuity, such as sequential continuity, normal (pointwise) continuity, uniform continuity, and some more. Furthermore the student learns that all of these notions coincide for functions $[0,1] \to \mathbb{R}$. However, the proofs that these notions are equivalent make use of non-constructive principles, making the extraction of algorithms from proofs often impossible. Working constructively one therefore constantly faces a choice between various competing notions. In this talk we will give an overview of how various schools of constructive mathematics have tried to deal with this problem and discuss the baggage that comes with the various attempts to simplify matters. We will also present some recent work showing that, surprisingly, in many situation assuming continuity is actually unnecessary from a constructive point of view.