Language Learning from a Computational Viewpoint

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Gold's Model

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IN

$16, 256, 16, 4$

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Gold's Model

IN $\longrightarrow$

$16, 256, 16, 4$

OUT $\longrightarrow$

"powers of 2"

Gold’s Model

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$16, 256, 16, 4,$
$4, 200, 24, 96$

OUT →

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Gold's Model

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M

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For every $t \in \mathbb{N}$:

$$\xrightarrow{\sigma_t := u_0 u_1 u_2 \ldots u_{t-1}} \quad M \quad \xrightarrow{h_t}$$

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For every $t \in \mathbb{N}$:

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Criteria for Success

For every presentation $u_0, u_1, \ldots, u_t, \ldots$ of L we gain

$$\xrightarrow{\sigma_0, \sigma_1, \ldots, \sigma_t, \ldots} \boxed{M \quad \xrightarrow{h_0, h_1, \ldots, h_t, \ldots}}$$

Definition

Let $b \in \mathbb{N}_{\geqslant 1} \cup \{*, \infty\}$. M $\mathbf{TxtFex}_b$-*learns* a language L if for every presentation T of L, i.e., $T \in \mathbf{Txt}(L)$, there is t_0 such that for all $t \geqslant t_0$ the hypothesis $h_t = M(T[t])$ is correct and $|\{M(T[t]) \mid t \geqslant t_0\}| \leqslant b$.
$b = 1$ is denoted by $\mathbf{TxtEx}$ (Gold) and $b = \infty$ by $\mathbf{TxtBc}$.

We are interested in collections of languages $\mathcal{L}$ such that there is a single M which $\mathbf{TxtFex}_b$-learns *every* language in $\mathcal{L}$.

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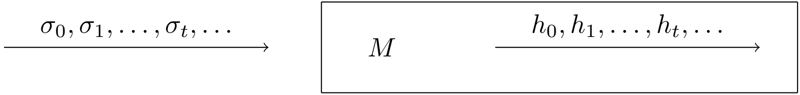

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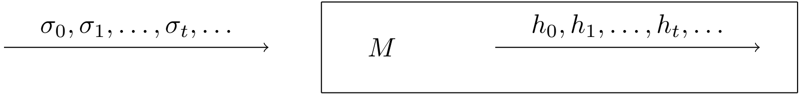

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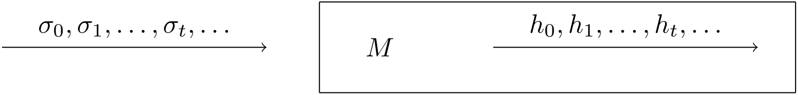

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Theorem (Case99)

For all $b \in \mathbb{N}_{\geqslant 1}$ *holds* $[\mathbf{TxtFex}_b] \subsetneq [\mathbf{TxtFex}_{b+1}]$ *and moreover* $\bigcup_{b \geqslant 1} [\mathbf{TxtFex}_b] \subsetneq [\mathbf{TxtFex}_*]$.

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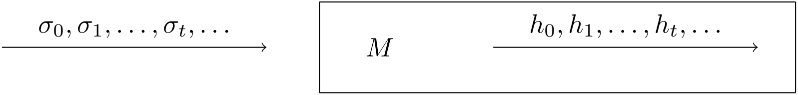

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Constraints on Learnability and Characterizations

Theorem (Subset Principle)

Let $b \in \mathbb{N}_{\geqslant 1} \cup \{*, \infty\}$ and M $\mathbf{TxtFex}_b$-learning the language L. Then there exists $D \subseteq L$ finite such that no proper subset L' of L with $D \subseteq L'$ is $\mathbf{TxtFex}_b$-learned by M.

Corollary

The set of all regular languages is not TxtBc-learnable.

Remark

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Insensitivity to Presentation Order

Definition

M is *set-driven* ($\mathbf{Sd}$) if for any two finite sequences with the same content it yields the same hypothesis.

Definition

$[\mathbf{SdTxtFex}_b]$ denotes the set of all $\mathcal{L}$ for which there exists a set-driven M $\mathbf{TxtFex}_b$-learning every language in $\mathcal{L}$.

The following result positively answers the question whether $[\mathbf{SdTxtEx}] \subsetneq [\mathbf{TxtEx}]$ carries over to $b > 1$.

Theorem (KSS17)

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Formal (Non-)U-Shaped Learning

Definition

M behaves *non-U-shaped* ($\mathbf{NU}$) on a presentation of a language L if whenever h_t and h_{t+n} are correct descriptions of L, then for all $i \leqslant n$ the description h_{t+i} is also correct.

Definition

M $\mathbf{TxtNUFex}_b$*-learns* a language L if it $\mathbf{TxtFex}_b$-learns L and additionally behaves non-U-shaped on every presentation of L.

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Results Regarding $\mathbf{NU}$-Learning

Theorem (CCJS08)

For all $b \in \mathbb{N}_{\geqslant 1} \cup \{*\}$ *holds* $[\mathbf{TxtNUFex}_b] = [\mathbf{TxtEx}]$.

Theorem (BCMSW05)

We have $[\mathbf{TxtEx}] \subsetneq [\mathbf{TxtNUBc}]$.

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Further results from CCJS08:

- $[\mathbf{TxtFex}_2] \subseteq [\mathbf{TxtNUBc}]$
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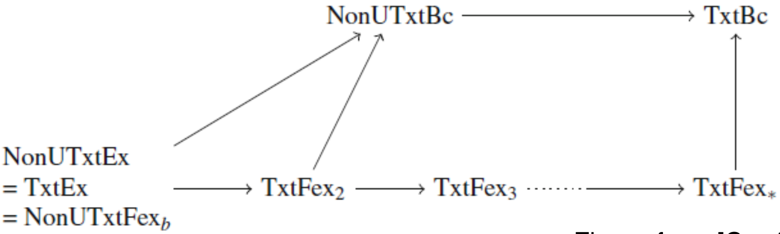
Map for $\mathbf{TxtNU}$-Learning



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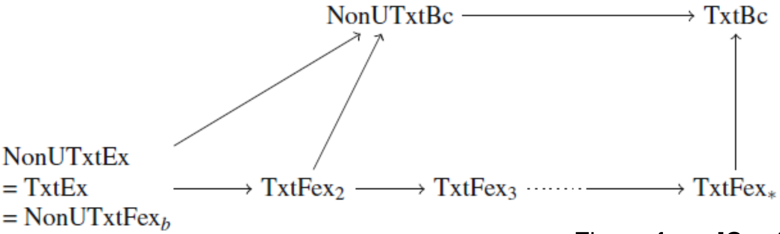

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Further Remarks

- Presenting positive and negative examples for L to the learner is covered by investigations on informants.
- Abundance of information and limited memory have been modelled by iterative learners and other attempts.
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Learning with Automata

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An automatic learner M uses two automatic functions hyp and mem which produce sequences $h_0, \ldots, h_t, \ldots$ interpreted as its hypotheses and $m_0, \ldots, m_t, \ldots$ standing for the information stored in M's memory via $h_{t+1} = \mathrm{hyp}(u_t, m_t)$ and $m_{t+1} = \mathrm{mem}(u_t, m_t)$.

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Thanks for listening! 

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