

On the Computability and Reducibility of Approximable Real Numbers

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A real number x is called computable if there is a computable sequence (x_s) of rational numbers which converges to x effectively. If the computable sequence is only increasing, then the limit x is called computably enumerable (c.e.). The arithmetical closure of the computable enumerable real numbers is the class of d.c.e. real numbers. Furthermore, the closure of d.c.e. real numbers under the total computable real functions is the class of divergence bounded computable (d.b.c.) real numbers. Finally, the limits of computable sequences of rational numbers without any convergence restriction are computably approximable real numbers. In this talk we are going to explore how the classes of d.c.e. and d.b.c. real numbers are related to different kind of reducibilities. In particular we will show that, a real number is d.c.e. iff it is Solovay reducible to a random c.e. real numbers, and it is d.b.c. iff it is convergence-dominated reducible to a random c.e. real number.