10. Exercise "Datenstrukturen und Effiziente Algorithmen", WS 18/19

Exercise 1: (5 Credits)
Use the `Extended_EUCLID` algorithm with input $a = 899$ and $b = 493$ to compute $gcd(a, b) = ax + by$ together with the coefficients x and y. Give the intermediate results for each recursive call.

Exercise 2: (7.5 Credits)
Suppose that the prime factorizations of a and b is $a = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$ and $b = p_1^{f_1} p_2^{f_2} \cdots p_r^{f_r}$, respectively, with zero exponents being used to make the set of primes $p_1, p_2, \ldots, p_r$ the same for both a and b. Show that

$$\gcd(a, b) = p_1^{\min\{e_1, f_1\}} p_2^{\min\{e_2, f_2\}} \cdots p_r^{\min\{e_r, f_r\}}.$$

Exercise 3: (7.5 Credits)
The i-th Fibonacci number is denoted by F_i, where $F_0 := 0$, $F_1 := 1$ and $F_i = F_{i-1} + F_{i-2}$, $i > 1$. Show that Fibonacci numbers can be used to convert miles to kilometers by:

$$F_n \text{km} \approx F_{n-1} \text{mi}.$$

Using the latter approximation, convert 100 miles to kilometers.

Exercise 4: (10 Credits)
What does the `Extended_EUCLID` algorithm return, if the input is $a = F_{k+1}$ and $b = F_k$? Prove your answer correct.

Deadline: Wednesday - January 16, 2019 - 12.15pm