Universität Greifswald
Institute für Mathematik and Informatik
Lecturer: Marc Hellmuth Tutor: Nikolai Nøjgaard

2. Exercise "Datenstrukturen und Effiziente Algorithmen", WS 18/19

Exercise 1: (2.5+2.5=5 Credits)
Let B a decision problem in the class $\mathcal{P}$ and A a decision problem with $A \leq_p B$.

(a) Explain in a few words if $A \notin \mathcal{NP}$ or $A \in \mathcal{NP}$.

(b) What can you conclude about the "subset relation" of the classes $\mathcal{P}$ and $\mathcal{NP}$ if Problem A is $\mathcal{NP}$-hard. Explain your results.

Exercise 2: "Subgraph Isomorphism" (10 Credits)
Show that the following decision problem is NP-complete:

> Given two undirected graphs H and G.
> Is there an isomorphism φ from H to some subgraph $G' \subseteq G$?

HINT: You may start the reduction from Clique.

Exercise 3: "BoolMatrixDecomposition" (15 Credits)
The following problem, called SetBasisProblem, is NP-complete:

> Given a set $\mathcal{C}$ of subsets of a finite set U and a positive integer k.
> Is there a set $\mathcal{B}$ of subsets of U with $|\mathcal{B}| \leq k$ such that for every $C \in \mathcal{C}$ there is a subset $\mathcal{B}_C \subseteq \mathcal{B}$ with $\cup_{B \in \mathcal{B}_C} B = C$?

Now, let A and B be binary matrices of dimensions $n \times k$ and $k \times m$, respectively. Let $C = A \circ B$ denote the ($n \times m$ matrix) Boolean product of A and B, i.e., the usual matrix product with the exception that $1 + 1 = 1$. By way of example

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

Use reduction from SetBasisProblem to show that the following problem BoolMatrixDecomposition is NP-complete.

> Given an $n \times m$ matrix C and a positive integer $k < \min\{m, n\}$.
> Is there a $n \times k$ matrix A and a $k \times m$ matrix B such that $C = A \circ B$?

Deadline: Wednesday - October 31, 2018 - 12.15pm