Universität Greifswald
Institute für Mathematik and Informatik
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7. Exercise "Datenstrukturen und Effiziente Algorithmen", WS 18/19

Exercise 1: (15 Credits)
Implement the depth-first-search (DFS) algorithm for directed graphs in adjacency list representation. Use this algorithm to decide, whether a given directed graph $G = (V, E)$ is cyclic, i.e., there is a simple path from $u \in V$ to $v \in V$, $u \neq v$ and an edge $(v, u) \in E$. Answer the following question doing simulations.

What is the approximate probability, that a random directed graph with $n = 10$ *nodes is cyclic, when for any pair of nodes* $u \neq v$ *there is an edge from* u *to* v *with probability* $p = 0.15$*, independent of all other edges?*

Self-loops $(u, u) \in E$ are not allowed. Simulate at least $m = 1000$ graphs and report the relative frequency of cyclic graphs.

Turn in your code and the output of your program. When handing in programming exercises, always document how to compile and run your program.

Exercise 2: (15 Credits)
A *strongly connected component* of a directed graph $G = (V, E)$ is an inclusion-maximal set of vertices $C \subseteq V$ such that for every pair of vertices u and v in C there is a path from u to v and from v to u.
Design a linear-time algorithm (pseudocode) to compute all strongly connected components of G. Proof the correctness and the run-time of your algorithm

HINTS:
You may use the DFS-algorithm and employ the graph $G^T = (V, E^T)$ where $E^T := \{(u, v) \colon (v, u) \in E\}$. You may also to prove the following statements to establish you algorithm:

(a) G and G^T have the same strongly connected components.

(b) Let C and C' be distinct strongly connected components in a directed graph G and let $u, v \in C$ and $u', v' \in C'$ and suppose that there is a path from u to u'. Then G contains no path from v' to v.

(c) Let $U \subseteq V$ and set $d(U) := \min\{u.d \colon u \in U\}$ and $f(U) := \max\{u.f \colon u \in U\}$. That is, $d(U)$ and $f(U)$ are the earliest discovery time and latest finishing time, respectively, of any vertex in U (cf. slide 10 in dfs.pdf).

Let C and C' be distinct strongly connected components in directed graph G. Suppose that there is an edge $(u, v) \in E$, where $u \in C$ and $v \in C'$. Then $f(C) > f(C')$.

Note, in the latter case we have $(v, u) \in E^T$.

You can use a different strategy than suggested here!

Deadline: Tuesday - December 11, 2018 - 10.15pm