Universität Greifswald
Institute für Mathematik and Informatik
Lecturer: Marc Hellmuth Tutor: Nikolai Nøjgaard

# 8. Exercise "Datenstrukturen und Effiziente Algorithmen", WS 18/19

**Exercise 1:** (7.5 Credits)
Let $G = (V, E)$ be a flow network with source $s$ and target $t$. In the lecture, we assumed that for all $v \in V$ there is a simple path $s \rightsquigarrow v \rightsquigarrow t$ from $s$ to $t$ that contains $v$. Suppose that $G$ violates this assumption and let $u$ be a vertex for which there is no path $s \rightsquigarrow u \rightsquigarrow t$. Show that there must exist a maximum flow $f$ in $G$ such that $f(u, v) = f(v, u) = 0$ for all $v \in V$.

**Exercise 2:** (7.5 Credits)
Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professor's house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of his town. Show how to formulate the problem of determining whether both his children can go to the same school as a maximum-flow problem.

**Exercise 3:** (7.5 Credits)
Suppose that both $f$ and $f'$ are flows in a network $G$ and we compute the augmented flow $f \uparrow f'$. Does the augmented flow satisfy the flow conservation property? Does it satisfy the capacity constraint?

**Exercise 4:** (7.5 Credits)
The *edge connectivity* of an undirected graph is the minimum number $k$ of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a simple cycle is 2.
Show how to determine the edge connectivity of an undirected connected graph $G = (V, E)$ by running a maximum-flow algorithm on at most $|V|$ flow networks (with bi-directional edges $(u, v)$, $(v, u)$ allowed), each having $O(|V|)$ vertices and $O(|E|)$ edges.

**Deadline: Wednesday - December 19, 2018 - 12.15pm**