Universität Greifswald
Institute für Mathematatik and Informatik
Lecturer: Marc Hellmuth Tutor: Nikolai Nøjgaard

9. Exercise "Datenstrukturen und Effiziente Algorithmen", WS 18/19

Exercise 1: (15 Credits)
Implementation maximum flow in random flow-networks:
Let $G = (V, E)$ be a random flow-network with $n = 20$ vertices $V = \{0, 1, \ldots, n-1\}$. Let $s = 0$ be the source of G and $t = n - 1$ be its target. (Directed) Edges (i, j) between vertices i and j are added with probality

$$p(i,j) = \begin{cases} 0 & \text{, if } j - i > 4 \vee j \leq i \\ 1 & \text{, if } (i = s \vee j = t) \wedge i < j \leq i + 4 \\ 1 - \frac{j-i-1}{4} & \text{, else.} \end{cases}$$

All (directed) edges in G have capacity 1.
Compute approximately the expected maximum flow in a random flow-network, by taking the average of the maximium flows for $m = 1000$ such random flow-networks $G_1, \ldots, G_m$. To compute the maximum flow use the Edmonds-Karp-Algorithm.
Hint:

- Use the Boost Graph Libary (BGL).
- The Edmonds-Karp-Algorithm is implemented in BGL. The file http://www.boost.org/doc/libs/1_65_1/libs/graph/example/edmonds-karp-eg.cpp gives you an example of how to use this algorithm. See also the BGL-Documentation for `edmonds_karp_max_flow`.

Exercise 2: (15 Credits)
A *perfect matching* is a matching in which every vertex is matched. Let $G = (V, E)$ be an an undirected bipartite graph with vertex partition $V = L \cup R$, where $|L| = |R|$. For any $X \subseteq V$, define the neighborhood of X as

$$N(X) := \{y \in V \mid \{x, y\} \in E \text{ for some } x \in X\},$$

that is, the set of vertices adjacent to some member of X.
Prove *Hall's marriage theorem:*
There exists a perfect matching in G if and only if $|A| \leq |N(A)|$ for every subset $A \subseteq L$.

XMAS - BonusCredits

The following exercises are optional and can be used in order to get additional credits. The content of the preceding exercises is still relevant for the exam, while the content of the bonus exercises are not .. HoHoHo:

I wish you a Happy Xmas and a Happy New Year!

Exercise X1: (10 Credits)
Santa Claus is a really scatterbrained person. After preparing all the presents for the children all over the world, he cached all presents in *one* of the n holy caves within the holy mountain nearby its little house. In order to protect all the presents he bewitched this particular cave and spoke with a deep voice:

HoHoHo – If anyone goes into the cave where the presents are, then this person gets a red nose and will become crazy after one week.

All other caves have not been enchanted. As said, Santa Claus is really scatterbrained and he forgot which of the caves contains the presents. Unfortunately, Xmas is in one week. Therefore, he forced his Xmas-helping-dwarfs to figure out where the cave is. As Santa Clause does not want to risk to have too many crazy dwarfs with red noses after their journey, he wants to keep the group of dwarfs that discover the holy mountain as small as possible.
Here the question: What is the minimal number of dwarfs that are needed to figure out where the present-cave is until Xmas? Describe which of the dwarfs has to go into which cave. The time needed to enter the cave and to come out is negligible.

Exercise X2: (10 Credits)
There are n hopeful children living in a small town and waiting for their Xmas-presents. Since Santa Claus is very bored by "always-the-same" poems recited by the children every year, he decided to give the children a riddle. If they cannot solve the riddle, then none of the children will get a present and xmas will become a NIGHTMARE for the children.

Here is the riddle: All n children will be lined up in a queue. All children have to look into the same direction such that every child can only see all the children in front of itself, but not the children behind. Moreover, all children will get a bobble-hat on their head. Each bobble-hat has one of $k \leq n$ colors $\{1, \ldots, k\}$. The children cannot see the color of their own hat and they are not allowed to take off their hat to see what color it is. After lining up all children, one after another has to say which color its own bobble-hat has. Moreover, after lining up, the children are not allowed to talk to each other and they are only allowed to say "color i", where $i \in \{1, \ldots, k\}$. Only one child is allowed to be wrong. If more than one child is wrong, then none of the children will get a present.

PLEASE SAVE XMAS!!!

You have the chance to talk to the children before they get lined up. Can you help to give the children a strategy such that they will *all* receive their presents? What would the strategy be? BTW: The children are quite smart and can follow your strategy.

Deadline: Wednesday - January 09, 2019 - 12.15pm