Universität Greifswald
Institute für Mathematik and Informatik
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10. Übung "Bioinformatik", SS 16

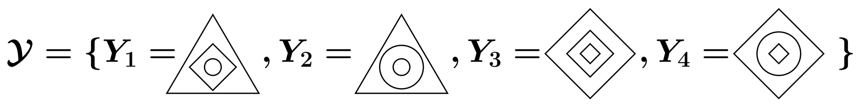
Aufgabe 1: (5 Credits)
[**Square Property.**] Let e and f be two incident edges of the Cartesian product graph $G_1 \Box G_2$ such that e and f are in different layers. Prove: Then there is exactly one diagonal-free square in $G_1 \Box G_2$ containing both e and f.

Aufgabe 2: (3 Credits)
Given the graph product $G_1 \star G_2$. Determine the number of edges and vertices of $G_1 \star G_2$ as a function of the number of edges and vertices of G_1 and G_2 for $\star \in \{\Box, \boxtimes, \times\}$.

Aufgabe 3: (4+4+9 = 17 Credits)
Let us assume aliens landed on earth, they can communicate via thought transmission and their bodies consist of three shells (outer-, middle-, and inner-shell). Scientists observed that genes that code for the aliens body consist of two nucleotides and determined the following accessible operations in the aliens geno- and phenotypespace:

Let $V(\mathbb{X}) = \mathbb{A}^2$ be the vertex set of the genotypespace $\mathbb{X}$ consisting of all genes of length two over the alphabet $\mathbb{A} = \{A, C, G, T\}$. Let $x = x_1x_2 \in V(\mathbb{X})$ be accessible from $y = y_1y_2 \in V(\mathbb{X})$, in symbols $x \curvearrowleft y$, iff for their Hamming distance holds $d_H(x, y) = 1$ and $x_1y_1 \notin \{AG, GA, CT, TC\}$. Let $(x, y) \in E(\mathbb{X})$ be an undirected edge in $\mathbb{X}$ if $x \curvearrowleft y$ or $y \curvearrowleft x$.

Let $\mathcal{Y}$ be the set of observable phenotypes of these alliens:



where each phenotype Y_i consists of the putative characters

- outer-shell (traits: Triangle, Diamont)
- middle-shell (traits: Circle, Diamont)
- inner-shell (traits: Circle, Diamont)

The vertex set $V(\mathbb{Y})$ of the phenotypespace $\mathbb{Y}$ is $\mathcal{Y}$. Let $f : V(\mathbb{X}) \to V(\mathbb{Y})$ be a given fitness function that maps genes $x = x_1x_2$ in the following way:

- $x \mapsto Y_1$ if $x_1 = A$,
- $x \mapsto Y_2$ if $x_1 = T$,
- $x \mapsto Y_3$ if $x_1 = C$,
- $x \mapsto Y_4$ if $x_1 = G$.

Let (Y_i, Y_j) be an undirected edge in $E(\mathbb{Y})$ iff $i \neq j$ and there is an edge (x, y) in $\mathbb{X}$ with $(f(x) = Y_i$ and $f(y) = Y_j)$ or $(f(y) = Y_i$ and $f(x) = Y_j)$.

(a) Give the undirected graph $\mathbb{X}$ as drawing or as edgelist.

(b) Draw the undirected graph $\mathbb{Y}$.

(c) Which of the putative characters are "real" characters and which of them are dependent if we assume to have a Cartesian product structure in the phenotype space. Prove your results.

Deadline: Monday - July 4st, 2016 - 2.00pm