Universität Greifswald
Institute für Mathematik and Informatik
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6. Exercise "Bioinformatics", SS 17

Aufgabe 1: (2+2+2+2+2=10 Credits)
Let $G = (V, E)$ be an undirected graph.

(a) Show that G has an even number of vertices with odd degree.

(b) Let $|V| \geq 2$. Show that G has two vertices of the same degree.

(c) Two edges $e = (a, b) \in E$ and $f = (x, y) \in E$ are in Relation Θ, in symbols $e\Theta f$, if and only if

$$d(a, x) + d(b, y) \neq d(a, y) + d(b, x).$$

Show that no two distinct edges on a shortest path in G are in relation Θ.

(d) Show that if G is a forest, then G has $|V| - |E|$ connected components.

(e) Assume now that $V = \{0, 1, 2, 3\}$ and $E = \{(0, 1), (1, 2), (1, 3)\}$. Show that $H \simeq G$ for $H = (\{a, b, c, d\}, \{(d, b), (a, d), (c, d)\})$. How many isomorphism are there?

Aufgabe 2: (1+1+1.5+1.5=5 Credits)
Let $G = (V, E)$ be an undirected graph, $\chi(G)$ denote the chromatic number and $\Delta(G)$ the maximum degree of G. The complement of $G = (V, E)$ is the graph $\overline{G} = (V, \overline{E})$ with $\overline{E} = \{\{u, v\} \mid \{u, v\} \notin E, u, v \in V, u \neq v\}$. Prove or disprove

(a) If G is bipartite then $\overline{G}$ is bipartite.

(b) There are graphs with $\chi(G) = \chi(\overline{G})$.

(c) If $\chi(G) = |V|$ then $\chi(G) = \Delta(G) + 1$.

(d) If $\chi(G) = \Delta(G) + 1$ then $\chi(G) = |V|$.

Aufgabe 3: (5 Credits)
A hypercube $Q_n = (V, E)$ is an undirected graph that consists of 2^n vertices such that each vertex $v \in V$ has a unique label

$$\ell(v) \in \{\text{binary number representing } k \mid 0 \leq k < 2^n\}.$$

The edge set E contains all edges $\{u, v\}$ for which the the Hamming distance between $\ell(v)$ and $\ell(u)$ is exactly 1. Show that Q_n is bipartite.

Deadline: Tuesday - May 16, 2017