Bioinformatics
(Intro Graphs)

Marc Hellmuth

Graphs and some (Basic-)Properties

- simple (un-)directed graphs $G = (V, E)$
- degree $\deg(v)$

Theorem

For any graph $G = (V, E)$ holds: $\sum_{v \in V} deg(v) = 2|E|$.

Theorem

Every graph contains an even number of vertices with odd degree.

Graphs and some (Basic-)Properties

- $\simeq$ isomorphism
- (induced) subgraph
- complete graphs K_n

Graphs and some (Basic-)Properties

- walk, trail, path, cycle
- connectedness

Graphs and some (Basic-)Properties

- forests and trees

Theorem

Let $G = (V, E)$ be an undirected graph. G is a tree if and only if for any two vertices $u, v \in V$ there is exactly on path connecting them.

Theorem

Let $G = (V, E)$ be an undirected connected graph.
G is a tree if and only if $|E| = |V| - 1$.

Corollary

Let $G = (V, E)$ be a connected undirected graph.
Then $|E| \geq |V| - 1$ and G has a spanning tree.

Graphs and some (Basic-)Properties

- distances

Lemma (Subpaths)

Any connected subgraph of a shortest path is a shortest path.

Lemma (Triangle Inequality)

Let $G = (V, E)$ be an undirected graph. For all $u, v, z \in V$ holds that

$$d(u, v) \leq d(u, z) + d(z, v)$$

Breadth-first search - `BFS(G,s)`

```
INPUT: Adjacencylist of a graph G = (V, E), source vertex s
for all v ∈ V do
    δ[v] = ∞, pred[v] = NIL
end for
δ[s] = 0, enqueue(Q,s)
while Q ≠ ∅ do
    u=dequeue(Q)
    for all v ∈adj(u) do
        if δ[v] = ∞ then
            δ[v] = δ[u] + 1
            pred[v] = u
            enqueue(Q,v)
        end if
    end for
end while
OUTPUT: Arrays δ, pred;
```

Breadth-first search - `BFS(G,s)`

Theorem

`BFS(G,s)` runs in time $O(|V| + |E|)$.

Theorem

After termination of `BFS(G,s)` we have $\delta(v) = d_G(s, v)$ for all $v \in V$

Graphs and Colorings

- (proper) coloring
- k-coloring and chromatic number $\chi(G)$

Theorem

For all graphs G holds: $\chi(G) \leq 1 + \Delta(G)$.

Graphs and Colorings

- bipartite graphs

Lemma

A graph G is bipartite if and only if G is 2-colorable.

Theorem

A graph G is bipartite if and only if G does not contain odd cycles.