UNIVERSITÄT GREIFSWALD
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Institute of Mathematics
and Computer Science

NOKO 2019

40th Northern German Colloquium on Applied Analysis and Numerical Mathematics

May 24-25, 2019

Universität Greifswald, Campus Beitz-Platz, Building C-Fun-Gene,
Felix-Hausdorff-Str.8, Konferenzraum A+B

noko2019@uni-greifswald.de

NOKO 2019

The annual Northern German Colloquium (NoKo) offers researchers in Northern Germany the opportunity to meet and to present their current research results in the area of Applied Analysis and Numerical Mathematics. NoKo particularly aims at enhancing collaboration and networking between the universities in the region.

Social program, May 24, 6:00 pm, guided tour in historic old town.
Meeting point: Stadtinformation am Markt.

Dinner, May 24, 7:00 pm at Restaurant Alter Fritz, Markt 13.
Each participant attending has to pay on his/her own.

NOKO 2019 Program

Friday, May 24

13:00	-	13:10	Opening
13:10	-	13:40	R. Mißfeldt, Lübeck A higher-order Runge-Kutta type scheme for the strong approximation for SPDEs
13:40	-	14:10	F. Bertrand, Berlin Equilibrated stress approximation and error estimation with application to solid mechanics
14:10	-	14:40	T. Richter, Magdeburg A parallel Newton-multigrid framework for 3d fluid-structure interactions
14:40	-	15:10	Coffee Break
15:10	-	15:40	K. Neymeyr, Rostock How nonnegative matrix factorizations can help to decompose spectral mixture data
15:40	-	16:10	M. Behr, P. Benner, J. Heiland, Magdeburg Differential Riccati Equation, A Galerkin Approach
16:10	-	16:40	K. Wicke, Greifswald Treebased networks -- an example of current research in mathematical phylogenetics
18:00	-	19:00	Guided Tour in Historic Old Town
19:00	-		Dinner at Restaurant “Alter Fritz”

Saturday, May 25

09:00	-	09:30	K. Mang, Hannover Challenges in phase-field modeling of fractures in incompressible solids
09:30	-	10:00	M. Eden, Bremen Deriving Effective Double-Poroelasticity Models for Heterogeneous Media
10:00	-	10:30	C. Mehlmann, Magdeburg A goal oriented error estimator for partitioned solution approaches in a finite element framework
10:30	-	11:00	Coffee Break
11:00	-	11:30	R. Görmer, Hannover Analysis suitable T-Splines in three space dimensions
11:30	-	12:00	M. Youssef, Greifswald Polynomial Interpolation Using Orthogonal Polynomials
12:00	-	12:30	F. Perner, Greifswald Dynamical Differential Games and their solution concepts with application on pursuit-evasion games
12:30	-	12:40	Closing

Book of Abstracts

A higher-order Runge-Kutta type scheme for the strong approximation for SPDEs

May 24
1:10pm

Ricarda Mißfeldt

University of Lübeck

This talk considers the numerical approximation of solutions of semilinear stochastic partial differential equations (SPDEs). A new derivative-free numerical scheme that converges strongly is proposed. It is based on a stochastic Taylor scheme introduced by S. Becker, A. Jentzen and P. Kloeden (2016) [1] and makes use of commutativity conditions on the noise term. A theoretical convergence analysis yields the strong order $3/2 - \epsilon$. Furthermore, some numerical simulation results are presented in the case of Nemytskij operators which are an important class in many applications.

References:

[1] Becker, S., Jentzen, A., and Kloeden, P. An exponential Wagner-Platen type scheme for SPDEs. SIAM J. Numer. Anal. (2016).

Equilibrated stress approximation and error estimation with application to solid mechanics

May 24
1:40pm

Fleurianne Bertrand

Humboldt University, Berlin

A stress equilibration procedure for linear elasticity is presented with emphasis on the behavior for nearly incompressible materials. It is based on the displacement-pressure approximation computed with a stable finite element pair and constructs an H(div)-conforming, weakly symmetric stress reconstruction. Thjst construction leads then to reconstructed stresses in the Raviart-thomas space which are weakly symmetric. The computation is performed locally on a set of vertex patches. The resulting error estimator constitute a guaranteed upper bound for the error with a constant that depends only on the shape regularity of the triangulation.

A parallel Newton-multigrid framework for 3d fluid-structure interactions

May 24
2:10pm

Thomas Richter[1], Lukas Failer[2]

[1]University of Magdeburg, [2]Technical University of Munich.

Fluid-structure interactions play a role in various application problems ranging from aeroelasticity to the flow of blood in the arteries. We consider the coupling of the incompressible Navier-Stokes equations with a nonlinear elasticity law. A problem common to hemodynamical applications (but also to applications in naval engineering) is the added mass effect, describing a very stiff coupling due to similar masses of solid and surrounding fluid. This effect calls for strongly coupled and implicit discretizations and solution schemes, so called monolithic approaches. A coupled description of both phases leads to very large, nonlinear and ill-conditioned algebraic problems. While direct solvers may be the method of choice in 2d, they are prohibitively expensive in 3d.

There has been some progress in literature, but all approaches suffer from the special type of the coupling. We will present a novel technique that is based on two simple ideas: 1st, we approximate the Jacobian by skipping some of the derivatives (with respect to the domain motion); 2nd, we condensate the solid's deformation and derive a coupled system in the velocities only. These two steps strongly reduce the problem size, but furthermore, the condition number of the Jacobian benefits, such that simple smoother within a multigrid are applicable.

We present results on different challenging test-cases and give studies on the parallel scalability (on shared-memory systems).

How nonnegative matrix factorizations can help to decompose spectral mixture data.

May 24
3:10pm

Klaus Neymeyr

Institut für Mathematik, Universität Rostock,
and Leibniz Institut für Katalyse, Rostock.

Nonnegative matrix factorization is a key tool for solving the blind source separation (BSS) problem. BSS has the aim to reconstruct the unknown nonnegative source terms from observable mixture data assuming a multilinear mixing model.

Here, we consider the bilinear Lambert-Beer "mixing" model in application to the spectral observation of a chemical reaction system. So-called multivariate curve resolution (MCR) methods serve to recover the pure component information from the spectral mixture data. However, MCR methods suffer from the intrinsic ambiguity of nonnegative matrix factorizations. A systematic approach is discussed for representing the sets of nonnegative factors in the form of the set of feasible solutions.

References:

[1] K. Neymeyr, M. Sawall, On the set of solutions of the nonnegative matrix factorization problem, SIAM J. Matrix Anal. Appl. 39, 1049-1069 (2018).

Software: http://www.math.uni-rostock.de/facpack/

Differential Riccati Equation, A Galerkin Approach

May 24
3:40pm

Maximilian Behr, Peter Benner, Jan Heiland

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We consider the differential Riccati equation,

$$\dot{X} = A^T X + XA - XBB^T X + C^T C.$$

The differential Riccati equation as well as the algebraic Riccati equation play important roles in applied mathematics like control theory and system theory. In our talk, we focus on the large-scale case. The numerical solution of these equation is challenging, in particular, because of the enormous amount of storage. A general approach, that has led to several algorithms, bases on an invariant subspace $Q \subseteq \mathbb{R}^{n \times n}$ such that $X(t) \in Q$ for all t. After identifying a suitable invariant subspace, we develop a Galerkin approach for the numerical solution of the differential Riccati equation. We review Davison-Maki methods for the numerical solution of the resulting equation.

Treebased networks – an example of current research in mathematical phylogenetics

May 24
4:10pm

Kristina Wicke

University of Greifswald

Phylogenetics is the study of the evolutionary history and relationships of different species. Originally a biological discipline, this area of research has given rise to a variety of mathematical and computational problems and has inspired an important branch of biomathematics: mathematical phylogenetics.

In this talk, I will shortly introduce the area of mathematical phylogenetics, before focusing on a very particular problem, namely the study of treebased networks. For a long time, so-called phylogenetic trees have been used to depict the evolutionary relationships among different species. However, there is more and more evidence that trees are not always an adequate representation of the evolutionary history of species, in particular if species undergo hybridization or horizontal gene transfer. Thus, phylogenetic networks have come to the fore as a generalization of phylogenetic trees. While trees and networks are in general different concepts (the latter may contain cycles, the former are acyclic), a special class of networks strongly related to trees – the class of treebased networks – has recently gained considerable interest in the literature. Roughly speaking, treebased networks are networks that can be obtained from phylogenetic trees by adding additional interior vertices and edges.

I will use the example of treebased networks to illustrate that many problems in mathematical phylogenetics are computationally hard, while there are polynomial-time algorithms for certain subproblems. To be precise, it has been shown that deciding whether a network is treebased or not is in general an NP-complete problem, and thus computationally hard. There are, however, certain classes of networks that are guaranteed to be treebased. In my talk, I introduce one of them – the class of edgebased networks – and show that edgebased networks can be recognized in linear time.

Challenges in phase-field modeling of fractures in incompressible solids

May 25
9:00am

Katrin Mang[1], Mirjam Walloth[2], Thomas Wick[1], Winnifried Wollner[2]

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Within this talk, fractures in incompressible materials are simulated by using a variational phase-field approach [2]. To simulate crack growth in incompressible materials, the standard model [1] is extended [3]. The approach builds on a mixed form of the solid displacement equation resulting in two unknowns. The fracture path is described with a phase-field function defined as a smoothed indicator variable. The crack irreversibility constraint is handled with a Lagrange multiplier. Thus, the final system consists of four variables: displacements, pressure, phase-field and the Lagrange multiplier. Numerical results of different mechanical tests are proposed to approve the quality of the new formulated model. These numerical tests include spatial mesh refinement studies and variations in Poisson's ratio approaching the incompressible limit. In addition, not just satisfying results but also challenges, drawbacks and future plans will be discussed.

References:

[1] D. Braess, Finite elements: Theory, fast solvers, and applications in solid mechanics, Cambridge University Press (2007).
[2] G.A. Francfort, J.-J. Marigo, Revisiting brittle fracture as an energy minimization problem, J. Mech. Phys. Solids, Volume 46/8 (1998) pages 1319-1342.
[3] K. Mang, T. Wick, W. Wollner, A phase-field model for fractures in incompressible solids, *arXiv* 1901.05378 (2019).

Deriving Effective Double-Poroelasticity Models for Heterogeneous Media

May 25
9:30am

Michael Eden

Universität Bremen

Biot's poroelasticity is a model aiming at describing the interaction between fluidflow and solid deformation within a porous medium. Extending this model to two-component media leads to the notion of double-poroelasticity.

In this talk, we consider highly heterogeneous two-component media composed of a poroelastic matrix (*fracture phase* with high porosity and high permeability) with periodically distributed inclusions that are itself poroelastic (storage phase with low porosity and low permeability). Real word examples motivating such a set-up are fissured rocks and stratified aquifers.

We show that the resulting coupled problem based on Biot's poroelasticity admits a unique weak solution and investigate the limit behavior of the solutions with respect to a scale parameter characterizing the heterogeneity of the medium. This is done in the context of the two-scale convergence method. By doing that, we arrive at an effective macroscopic model in the form of a PDE with an additional delay term.

A goal oriented error estimator for partitioned solution approaches in a finite element framework.

May 25
10:00am

C. Mehlmann, T. Richter

Otto-von-Guericke-Universität Magdeburg

Subject of this talk is the derivation and the numerical analysis of a *goal oriented error estimator for partitioned solution approaches*. The error estimator is applicable for a general class of non-stationary differential equations that are coupled to a transport process and solved with a partitioned solution approach such as Burgers equation

$$\begin{aligned} \partial_t \mathbf{v}(1+20h) + \epsilon\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = \mathbf{f}, \quad \mathbf{v}(0) = \mathbf{v}^0, \\ \partial_t h + \operatorname{div}(h\mathbf{v}) + \epsilon_t \Delta h = g, \quad h(0) = h^0, \end{aligned} \tag{1}$$

on $\Omega = (-1,1)^2$, with $\epsilon = 0.01$ and $\epsilon_t = 0.1$.

The error estimator is able to approximate the different error contributions that occur in measuring a functional of interest. It consists of the *spatial* and *temporal discretization error* as well as the *splitting error* due to the use of a partitioned solution approach. The error estimator is based on the *dual weighted residual method* introduced by Becker and Rannacher [1]. Based on the approximation of the error estimator, we derived an *adaptive refinement strategy* that balances the different error contribution to reduce the overall numerical error.

We evaluated the error estimator on model (1) and the viscous-plastic sea ice model which includes a strong nonlinear momentum equation and transport process.

References:

[1] R. Becker, R Rannacher, An Optimal Control Approach to A Posteriori Error Estimation in Finite Element Methods,Acta Numerica 2001, 10, 1-102.

Analysis suitable T-Splines in three space dimensions

May 25
11:00am

Robin Görmer

Leibniz Universität Hannover

Isogeometric analysis is a method for solving partial differential equations with higher degree polynomial basis functions, developed in recent years. Usually, NURBS are used for modelling. However, since adaptive refinement yield unstructured meshes, the concept of T-Splines was introduced. In this setting we can only use special meshes for numerical analysis and implementation, called analysis suitable meshes. In 2D, a practical definition of analysis suitability was made and approximation properties were proven, see [1]. This definition uses the indices of the knot vectors used for T-Splines. For arbitrary space dimension d, another definition based on the support of T-Splines was introduced, see [2]. Implementation requires additional memory, but approximation theory still holds. We will extend the definition from [1] to 3D in a first step and show then the equivalence of the existing definitions under the assumptions of no knot repetitions.

References:

[1] L. Beirão da Veiga et al. (2014), Mathematical analysis of variational isogeometric methods". In: Acta Numerica 23, pp. 157-287.
[2] P. Morgenstern (2016), Globally Structured Three-Dimensional Analysis-Suitable T-Splines: Definition, Linear Independence and m-graded local refinement". In: SIAM Journal on Numerical Analysis 54.4, pp. 2163-2186.

Polynomial Interpolation Using Orthogonal Polynomials

May 25
11:30am

Maha Youssef

Institute of Mathematics and Computer Science, University of Greifswald

In this talk we investigate polynomial interpolation using orthogonal polynomials. We use weight functions associated to orthogonal polynomials to define a weighted form of Lagrange interpolation [1]. We introduce an upper bound of error estimation for such kind of approximations. Later, we introduce the sufficient condition of Stenger's conjecture for orthogonal polynomials and numerical verification for such conjecture [2].

References:

[1] M. Youssef, G. Baumann, "Lagrange Interpolation Using Orthogonal Polynomials, Stenger's Conjecture, Numerical Approach", (2019), preprint.
[2] F. Stenger, G. Baumann, V. Koures, "Computational Methods for Chemistry and Physics, and Schrödinger in 3 + 1", Advances in Quantum Chemistry 71, pp. 265-298

Dynamical Differential Games and their solution concepts with application on pursuit-evasion games

May 25
12:00pm

Florian Perner

Institute of Mathematics and Computer Science, University of Greifswald

With this talk I present an approach for differential game problems which generalize the optimal control theory. In comparison to optimal control in differential game theory two or more players attempt to control the state of a dynamical system given by ordinary differential equations in order to minimize their very own cost functional. Searching for a saddle point is related closely with finding a solution for these games in terms of Nash Equilibria from the game theoretical point of view. Even with a small number of variables this might become computationally difficult. Some thoughts on algorithms for both the state-constrained and unconstrained case are given. The results are shown for pursuit-evasion games as an application for a differential game with two players.